• Am I arranging non-distinct objects?
(Am I arranging objects in which some of the
objects are identical?)
Use \[
\frac{n!}{n_1!n_2! \cdots n_r!}
\]
• Am I arranging distinct objects?
(Am I arranging objects that all look different?)
Draw blanks and use multiplication principle
• Am I selecting a subset out of the group?
Use combinations

Fall 2011
Math 141 Week-in-Review #6
Exam 2 Review
courtesy: Kendra Kilmer
(covering Sections 3.1-3.3, 6.1-6.4, and 7.1)
(Please note that this review is not all inclusive)

1. A quiz consists of four multiple choice questions (that each have five choices) and five true/false questions. How many ways can a student answer the exam if a penalty is imposed for a wrong answer (thus, a student may choose to leave the question blank)?

\[
\frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{5} = 314928
\]
2. Nick, an enthusiast of books, is doing some cleaning. He has found that he has two identical copies of *The Never Ending Story*, five identical copies of *There and Back Again*, and three identical copies of *Dreamcatcher*. How many different ways can he arrange these books on his shelf?

\[
\frac{10!}{2!5!3!} = 2520
\]
3. A shipping carton has 30 games in it. If it is known that there are 8 defective games in the carton, how many ways can you select 4 games and get exactly 1 defective game?

\[
\binom{8}{1} \cdot \binom{22}{3} = 12,320
\]
Johnny has been chosen to be one of the team captains for the weekly kickball game. There are 19 other kids in his class and he must choose 9 of them to be on his team. If he is good friends with 4 of the kids, how many ways can he choose his team so that he has at least 2 of his good friends on his team?

\[
\begin{align*}
\text{19} & \quad \text{15 others} \\
\text{4 good friends} & \quad \text{select 9} \\
\end{align*}
\]

at least 2 good friends
exactly 2 good friends
exactly 3 good friends
exactly 4 good friends

\[
\begin{align*}
\binom{4}{2} \cdot \binom{15}{7} & \quad \text{26F others} \\
\binom{4}{3} \cdot \binom{15}{6} & \quad \text{36F others} \\
\binom{4}{4} \cdot \binom{15}{5} & \quad \text{46F others} \\
\end{align*}
\]

= 16333
5. A National Honor Society Club consists of a president, vice-president, 3 treasurers, and 2 secretaries. The president and vice-president are to be chosen from 6 candidates, the 3 treasurers from 7 candidates, and the 2 secretaries from 12 candidates. How many different groups can be formed?

\[
\frac{C(6,1) \cdot C(5,1)}{P \cdot VP} \cdot \frac{C(7,3)}{T} \cdot \frac{C(12,2)}{S} = 69,300
\]
6. Susie has five different pieces of fruit, eight different vegetables, and seven different cookies. If she is going to pack a lunch that contains five items, how many ways can she have exactly two pieces of fruit or exactly three cookies?

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]

\[ \frac{C(5,2) \cdot C(13,3) + C(7,3) \cdot C(13,2)}{2F \cdot 3Others} - \frac{C(5,2) \cdot C(7,3) \cdot C(8,0)}{2F \cdot 3C \cdot 0Other} \]

\[ \begin{align*}
20 & \quad 20 & \quad 20 \\
5F & \quad 15Others & \quad 7C & \quad 13Others \\
select S & \quad select S & \quad select S & \quad select S \\
\end{align*} \]

\[ = 6930 \]
7. How many different ten digit numbers can be formed from three 2's, one 7, five 8's, and a 9?

arranging objects in which some are identical

3 twos
1 seven
5 eights
1 nine

10 total

\[
\frac{10!}{3!1!5!1!} = 5040
\]
8. An experiment consists of randomly choosing 5 paper clips from a box containing 12 green paper clips, 9 red paper clips, and 3 blue paper clips. In how many ways can this be done if exactly 3 of the paper clips selected are the same color?

\[
\begin{align*}
&\binom{12}{3} \cdot \binom{9}{2} + \binom{9}{3} \cdot \binom{12}{2} + \binom{3}{3} \cdot \binom{21}{2} = 23550 \\
\end{align*}
\]
9. How many four digit numbers can be formed from the digits 0, 1, 3, 4, 5, 7, 8 if each number formed must be even, repetition of digits is not allowed, and the first digit must be a non-zero number?

\[
\frac{6 \cdot 5 \cdot 4 \cdot 1}{7!} + \frac{5 \cdot 5 \cdot 4 \cdot 2}{7!} = 120 + 200 = 320
\]
10. Determine graphically the solution set for the following system of linear inequalities. Label all corner points.

- $y \leq -x + 4$
- $y = -x + 4$
- $y = -2x + b$
- $x + y \leq 4$
- $2x + y \leq 6$
- $2x - y > -1$
- $x \geq 0, y \geq 0$

**Graph:**
- $y = -x + 4$
- $y = -2x + b$
- $x = 3$
- $y = 2x + 1$
- $y = -2x - 1$
- $x = 2x + 1$

**Points:**
- $(10, 1)$
- $(3, 0)$
- $(2, 2)$
- $(1, 2)$
- $(0, 0)$
11. Glen Hill Inc. produces three kinds of shampoos. It takes 2.5 hours to produce 1,000 bottles of formula I, 3 hours to produce 1,000 bottles of formula II, and 4 hours to produce 1,000 bottles of formula III. The profit for each 1,000 bottles of formula I, formula II, and formula III are $180, $200, and $300 respectively. Suppose we want to production run, there are enough ingredients on hand to make at most 5,000 bottles of formula I, 12,000 bottles of formula II, and 8,000 bottles of formula III. Furthermore, suppose the time for the production run is limited to a maximum of 70 hours. How many bottles of each formula should be produced in order to maximize the profit? Set up the linear programming problem but DO NOT SOLVE.

\[
\begin{align*}
\text{Maximize} \quad P &= 180x + 200y + 300z \\
\text{Subject to} \quad 2.5x + 3y + 4z &\leq 70 \\
0 &\leq x \leq 9 \\
0 &\leq y \leq 12 \\
0 &\leq z \leq 6
\end{align*}
\]
12. Given the following linear programming problem:

Maximize \[ P = 10x + 2y \]
Subject to \[
\begin{align*}
x + y &\leq 12 \\
2x + y &\leq 16 \\
x &\geq 0, y &\geq 0
\end{align*}
\]

The optimal solution occurs at the intersection of which two lines?

The optimal solution occurs at the intersection of \( y = -2x + 16 \) and the \( x\)-axis (\( y = 0 \)).
13. Using the following sets, determine whether each statement is True or False.

\[ U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \]
\[ A = \{2, 4, 9\}, B = \{1, 3, 10\}, C = \{5, 9, 10\} \]
\[ B^c = \{2, 4, 5, 6, 7, 8, 9\} \]
\[ C^c = \{1, 2, 3, 4\} \]
\[ n(A \cup C) = 2 \]

(a) \( 5 \in B^c \)  **True**

(b) \( \{1, 2\} \subseteq ((A \cap C^c) \cup B) \)
\[ \{1, 2\} \subseteq \{1, 2, 3, 4, 10\} \]
**True**

(c) \( B \cup C \) has no proper subset
**False**

(d) \( (A \cup B)^c \cap C = A^c \cap B^c \cap C \)
Apply De Morgan's Law to
\[ (A \cup B)^c \cap C = A^c \cap B^c \cap C \]
**True**

(e) \( 3 \in (B \cap C) \)
**False**

(f) \( n(A \cup C^c) = 2 \)
\[ A \cup C^c = \{1, 2, 3, 4, 6, 7, 8, 9\} \]
\[ n(A \cup C^c) = 8 \]
14. A survey was conducted with 200 freshmen in college to learn of their vegetable preferences. Use a Venn Diagram to represent the following information:

- 60 freshmen will only eat broccoli
- 60 freshmen will eat exactly 2 of the vegetables
- 95 freshmen will NOT eat spinach
- 9 freshmen will eat all 3 vegetables
- 63 freshmen will eat okra
- 36 freshmen will eat okra and broccoli
- 12 freshmen will only eat okra and spinach

How many freshmen will eat exactly one of these vegetables?

\[ 60 + 3 + 15 = 78 \]
15. Shade the following set on a three-circle Venn Diagram: \((A \cup B^c) \cap C\)
16. An experiment consists of rolling a fair six-sided die and drawing one marble out of a box containing 3 red and 7 blue marbles.

(a) Find the sample space associated with the experiment.
\[ S = \{ 1R, 2R, 3R, 4R, 5R, 6R, 1B, 2B, 3B, 4B, 5B, 6B \} \]

(b) Find the event, \( E \), that an even number is rolled.
\[ E = \{ 2R, 4R, 6R, 2B, 4B, 6B \} \]

(c) Find the event, \( F \), that a green marble is selected.
\[ F = \emptyset \]

(d) Find the event, \( G \), that a number less than three is rolled.
\[ G = \{ 1R, 2R, 1B, 2B \} \]

(e) Are the events \( E \) and \( F \) mutually exclusive?

\[ E \cap F = \emptyset \quad \text{Yes} \]

(f) Are the events \( E \) and \( G \) complementary?

\[ E^c = \{ 1R, 3R, 5R, 1B, 3B, 5B \} \quad \text{\( \supseteq \)} \quad G = \{ 1R, 2R, 1B, 2B \} \quad \text{No} \]

(g) How many events does this experiment have?
\[ \# \text{ of events} = 2^2 \]
\[ = 2^{12} = 4096 \]