Sections 7.5 and 7.6

- If A and B are events in an experiment and $P(A) \neq 0$, then the conditional probability that the event B will occur given that the event A has already occurred is $\frac{P(A \cap B)}{P(A)}$ or $P(B|A) = \frac{P(AB)}{n(A)}$.

- **Product Rule:** $P(A \cap B) = P(A) \cdot P(B|A)$

- **Tree Diagrams**

- If A and B are independent events then $P(A|B) = P(A)$ and $P(B|A) = P(B)$. Furthermore, two events are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$. This can be extended to more than two events.

- **Bayes' Theorem**

\[
P(A|B) = \frac{P(A\cap B)}{P(B)}
\]
1. A survey was done of students in which both their age and the number of pieces of candy they ate on Halloween was observed. The following results were obtained:

<table>
<thead>
<tr>
<th>Age Group</th>
<th>0</th>
<th>1 – 10</th>
<th>&gt; 10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
<td>5</td>
<td>30</td>
<td>45</td>
<td>80</td>
</tr>
<tr>
<td>18 – 20</td>
<td>20</td>
<td>50</td>
<td>40</td>
<td>110</td>
</tr>
<tr>
<td>Over 20</td>
<td>25</td>
<td>75</td>
<td>60</td>
<td>160</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>155</td>
<td>145</td>
<td>350</td>
</tr>
</tbody>
</table>

(a) What is the probability that a student who ate 10 or fewer pieces of candy was over 20?

\[
P(\text{Over } 20 \mid \text{at most 10}) = \frac{n(\text{Over 20 and at 10 or fewer})}{n(\text{at 10 or fewer})} = \frac{25+75}{50+155} = \frac{100}{205} = \frac{20}{41}
\]

(b) What is the probability that a student under 18 ate more than 10 pieces of candy?

\[
P(>10 \mid \text{under 18}) = \frac{n(>10 \cap \text{under 18})}{n(\text{under 18})} = \frac{45}{80} = \frac{9}{16}
\]

(c) What is the probability that a student ate 10 pieces of candy or less given they were in the age group 18 – 20?

\[
P(\text{10 pieces or less} \mid 18-20) = \frac{n(\text{10 pieces or less} \cap 18-20)}{n(18-20)} = \frac{20+50}{110} = \frac{70}{110} = \frac{7}{11}
\]
2. If two fair six-sided dice are rolled, what is the probability that the sum is 8 given that the sum is greater than 7?

\[ P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{5}{15} = \frac{1}{3} \]

OR

\[ P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{5/36}{5/36} = \frac{5}{36} \cdot \frac{36}{15} = \frac{1}{3} \]
3. In one area, 4% of the population drive luxury cars. However, 17% of the CPAs drive luxury cars. Are the events "person drives a luxury car" and "person is a CPA" independent?

\[ P(L) = 0.04 \]
\[ P(L|C) = 0.17 \]

If \( L \mid C \) are independent,
\[ P(L|C) = P(L) \]
\[ 0.17 \neq 0.04 \]

Thus, the events are not independent.
4. In a home theater system, the probability that the video component needs repair within 1 year is 0.02, the probability that the electronic component needs repair within 1 year is 0.03, and the probability that the audio component needs repair within 1 year is 0.01. Assuming the probabilities are independent, what is the probability that exactly two components will need repair within 1 year.

Let $V$ be the event the video comp needs repair within 1 year.

$E$ "" electronic "" "" ""

$A$ "" audio "" "" 

$P(V) = 0.02 \quad P(V^c) = 1 - 0.02 = 0.98$

Exactly 2 components need repair

$P(E) = 0.03 \quad P(E^c) = 0.97$

$P(A) = 0.01 \quad P(A^c) = 0.99$

= video's electronic need repair but audio doesn't

= video's audio "" "" electronic doesn't

= or electronic's audio "" "" video doesn't

= $P(V \cap E \cap A^c) + P(V \cap A \cap E^c) + P(E \cap A \cap V^c)$

= $P(V) \cdot P(E) \cdot P(A^c) + P(V) \cdot P(A) \cdot P(E^c) + P(E) \cdot P(A) \cdot P(V^c)$

= $(0.02)(0.03)(0.99) + (0.02)(0.01)(0.97) + (0.03)(0.01)(0.98)$

= 0.001082

OR/

OR/

= $0.02 \quad 0.03 \quad 0.01$

= $0.98 \quad 0.97 \quad 0.91$

= (0.02)(0.03)(0.99) + (0.02)(0.97)(0.01)

= 0.001082
5. Unfortunately, two lights have been added to Mrs. Kilmer’s commute to work. After a few weeks of collecting data, she has determined that the probability that she will be stopped by the light at 2818 and Holleman is 0.2857 whereas the probability that she will be stopped by the light at 2818 and Luther is 0.0476. Assuming these events are independent, what is the probability that she will be stopped by at least one of the two lights on her drive to work tomorrow morning?

Let \( H \) be the event she is stopped at Holleman

Let \( L \) be the event she is stopped at Luther

\[
P(H) = 0.2857 \\
P(L) = 0.0476
\]

\[
P(H \cup L) = P(H) + P(L) - P(H \cap L)
\]

\[
= (0.2857) + (0.0476) - P(H) \cdot P(L)
\]

\[
= 0.2857 + 0.0476 - (0.2857)(0.0476)
\]

\[
\approx 0.3197
\]
6. A particular rocket assembly depends on two components, A and B. If either is working properly, the assembly will function, whereas if both fail, the assembly will not function. Assume that the functioning of either component is independent of the other, that the probability of failure of A is 0.002, and that the probability of failure of B is 0.01.

(a) What is the probability that both components will fail simultaneously?

Let A be the event that A is functioning properly.

\[ P(A^c \cap B^c) = P(A^c) \cdot P(B^c) = (0.002)(0.01) = 0.00002 \]

(b) What is the probability that at least one of the components is working properly?

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

\[ = 0.998 + 0.99 - (0.998)(0.99) = 0.99998 \]

(c) What is the probability that one component will be functioning properly and one will not?

\[ P(A \cap B^c) + P(B \cap A^c) \]

\[ = P(A) \cdot P(B^c) + P(B) \cdot P(A^c) \]

\[ = (0.998)(0.01) + (0.99)(0.002) = 0.01196 \]
7. A survey has shown that 95% of the women in a certain community work outside the home. Of these women, 56% are married, while 85% of the women who do not work outside the home are married. Find the probabilities that a woman in the community can be categorized as follows:

(a) A single woman working outside the home.
(b) Married.

Let \( W \) be the event that they work outside of the home.
Let \( M \) be the event they are married.

\[
\begin{align*}
& P(W) = 0.95 \\
& P(M | W) = 0.56 \\
& P(M | W^c) = 0.85
\end{align*}
\]

\[
\begin{align*}
a) \quad & P(M^c \cap W) = P(W \cap M^c) = (0.95)(0.44) = 0.418 \\
b) \quad & P(M) = (0.95)(0.56) + (0.05)(0.85) = 0.5745
\end{align*}
\]
8. A bicycle factory runs two assembly lines, A and B. If 95% of line A’s products pass inspection, while only 90% of line B’s products pass inspection, and 60% of the factory’s bikes come off assembly line B (the rest off A), find the probability of a bike not passing inspection.

Let A be event the bike comes from assembly line A.
Let B be event the bike passes inspection.
Let P be event the bike does not pass inspection.

\[ P(A) = 0.4, \quad P(B) = 0.6, \quad P(\text{not } B) = 0.4 \]

\[ P(P|A) = 0.95, \quad P(P|B) = 0.9 \]

\[ P(P) = P(P|A)P(A) + P(P|B)P(B) = (0.95)(0.4) + (0.9)(0.6) = 0.86 \]

\[ P(\text{not } P) = 1 - P(P) = 0.14 \]
9. Given the following tree diagram, answer the following questions:

(a) \( P(A \cap D) = (.1)(.15) = .015 \)

(b) \( P(A) = .1 \)

(c) \( P(B \cup F) = P(B) + P(F) - P(B \cap F) \)
\[ = .2 + \left[ (.1)(.3) + (.2)(.45) + (.7)(.25) \right] - (.2)(.45) \]
\[ = .405 \]

(d) \( P(D|A) = .15 \)

(e) \( P(C|F) = \frac{P(C \cap F)}{P(F)} = \frac{(.7)(.25)}{(.1)(.3) + (.2)(.45) + (.7)(.25)} = \frac{35}{89} \)
10. A crate contains 8 basketballs and 5 footballs. A bag contains 6 basketballs and 9 footballs. An experiment consists of selecting a ball at random from the crate and placing it in the bag. A ball is then randomly selected from the bag. If a football is selected from the bag, what is the probability that the transferred ball was a football?

\[
P(F_1 | F_2) = \frac{P(F_1 \cap F_2)}{P(F_2)}
\]

\[
= \frac{(8/13)(9/16)}{(8/13)(9/16) + (5/13)(10/16)}
\]

\[
= \frac{25/61}{1}
\]
11. The probability that a person with certain symptoms has a disease is 0.8. The blood test used to confirm this diagnosis gives positive results for 90% of people with the disease and 5% of those without the disease. What is the probability that an individual who has the symptoms and who reacts positively to the test actually has the disease?

\[
P(D) = 0.8 \\
P(+|D) = 0.9 \\
P(+|D^c) = 0.05
\]

\[
P(D|+) = \frac{P(D \cap +)}{P(+)} = \frac{(0.8)(0.9)}{(0.8)(0.9) + (0.2)(0.05)} = \frac{72}{73}
\]
12. A building contractor buys 70% of his cement from supplier A, and 30% from supplier B. A total of 90% of the bags from A arrive undamaged, while 95% of the bags from B arrive undamaged. Find the probability that a damaged bag came from supplier B.

\[
\begin{align*}
\text{P}(A) &= 0.7 \\
\text{P}(B) &= 0.3 \\
\text{P}(D^c | A) &= 0.9 \\
\text{P}(D^c | B) &= 0.95
\end{align*}
\]

\[
P(B | D) = \frac{\text{P}(B \cap D)}{\text{P}(D)}
\]

\[
= \frac{0.3 \cdot 0.05}{0.7 \cdot 0.1 + 0.3 \cdot 0.05}
\]

\[
= \frac{3}{17}
\]
The following table shows the proportion of people over 18 who are in various age categories, along with the probabilities that a person in a given age category will vote in a general election.

<table>
<thead>
<tr>
<th>Age</th>
<th>Percent of Voting Age Population</th>
<th>Probability of a Person of this Age Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 – 21</td>
<td>11.0</td>
<td>0.48</td>
</tr>
<tr>
<td>22 – 24</td>
<td>7.6</td>
<td>0.53</td>
</tr>
<tr>
<td>25 – 44</td>
<td>37.6</td>
<td>0.68</td>
</tr>
<tr>
<td>45 – 64</td>
<td>28.3</td>
<td>0.64</td>
</tr>
<tr>
<td>65 or over</td>
<td>15.5</td>
<td>0.74</td>
</tr>
</tbody>
</table>

(a) Find the probability that a voter is in the age category 18 – 21.

(b) Find the probability that a person who did not vote was in the age category 25 – 44.