Section 3.1

- To graph a system of linear inequalities:
  - Solve all inequalities for $y$.
  - Graph the corresponding equations. If the inequality is a strict inequality ($<$ or $>$) draw the line as a dotted line to represent that the points on the line are NOT part of the solution. If the inequality is inclusive ($\leq$ or $\geq$) draw the line as a solid line to represent that the points on the line are part of the solution.
  - Determine which half-plane satisfies the inequality. You can do this by using a test point or by using the following “rules”. Please note that these “rules” only work once we have the inequality solved for $y$.
    - If you have $y \leq f(x)$ or $y < f(x)$ the solution set lies below the line.
    - If you have $y \geq f(x)$ or $y > f(x)$ the solution set lies above the line.
  - Our solution set (feasible region) consists of all points that satisfy all of the inequalities.

- A solution set is **bounded** if all of the points in our solution set can be enclosed by a circle. Otherwise, we say that the solution set is **unbounded**.
- A corner point is a point along the boundary of our feasible region in which we have a sharp turn.

*Keep in mind that if you multiply or divide by a negative number, the inequality flips.*
1. Graph the following systems of linear inequalities. State whether the region is bounded or unbounded.

(a) \( x + y > 5 \)  
\( y \leq 8 \)

Graph \( y = 8 \)

Graph \( y = -x + 5 \)

\[
\begin{align*}
 y &= -x + 5 \\
 y &= -5 \\
 y &= 5 \\
(0, 5) \\
(5, 0)
\end{align*}
\]

(b) \( 20x + 10y \geq 100 \)  
\( 10x + 20y \geq 100 \)  
\( 10x + 10y \geq 80 \)

x \geq 0, y \geq 0

Graph \( y = -2x + 10 \)

Graph \( y = -\frac{1}{2}x + 5 \)

\[
\begin{align*}
 y &= -2x + 10 \\
 x &= 10 \\
 y &= -\frac{1}{2}x + 5 \\
(0, 10) \\
(10, 0)
\end{align*}
\]

Graph \( y = -x + 8 \)

Graph \( y = -\frac{1}{2}x + 5 \)

\[
\begin{align*}
 y &= -x + 8 \\
 x &= 10 \\
 y &= -\frac{1}{2}x + 5 \\
(0, 10) \\
(10, 0)
\end{align*}
\]
Section 3.2

- A Linear Programming Problem consists of a linear objective function to be maximized or minimized subject to constraints in the form of linear equations or inequalities.
- When setting up a linear programming problem, make sure to precisely define your variables, include your objective (maximize or minimize) and objective function along with all of your constraints.

2. A manufacturer of baby formula makes two types of formula: Type A and Type B. One container of type A requires 30 grams of whey, costs $3 to make, and sells for $25. One container of type B requires 20 grams of whey, costs $3.50 to make, and sells for $28. If the manufacturer has 4000 grams of whey available, $500 available to produce the formula, and would like to maximize their revenue, how many containers of each type of formula should they make? (Set up but DO NOT SOLVE the linear programming problem.)

\[
\begin{align*}
\text{Maximize} & \quad R = 25x + 28y \\
\text{Subject to} & \quad 30x + 20y \leq 4000 \\
& \quad 3x + 3.5y \leq 500 \\
& \quad x \geq 0, \quad y \geq 0
\end{align*}
\]
3. A small city is trying to determine the best way to raise funds in a given month. One thing they know for certain is that they want to issue at least 500 citations. They have determined that issuing a class C misdemeanor will require 20 minutes of processing time but will yield a revenue of $500. Issuing a class B misdemeanor will require 30 minutes of processing time but will yield a revenue of $1,000. Issuing a class A misdemeanor will require 1 minute of processing time but will yield a revenue of $2,000. If they have 60 hours available to process these misdemeanors and have decided that they want to issue at least three times as many class C misdemeanors as class A and class B misdemeanors combined, how many citations of each category should they issue in order to maximize their revenue? (Set-up but DO NOT SOLVE the linear programming problem.)

\[
\begin{align*}
    x &= \text{# of class C citations issued} \\
    y &= \text{# of class B citations issued} \\
    z &= \text{# of class A citations issued}
\end{align*}
\]

Maximize \( R = 500x + 1000y + 3000z \)

Subject to:

\[
\begin{align*}
    x + y + z &\geq 500 \\
    20x + 50y + 90z &\leq 3600 \\
    x &\geq 3(z + y) \\
    x \geq 0, y \geq 0, z \geq 0
\end{align*}
\]
Section 3.3

- If a linear programming problem has a solution, we are guaranteed that it will occur at a corner point of the feasible region.
- If the feasible region is bounded then the objective function has both a maximum and minimum value.
- If the feasible region is unbounded and the coefficients of the variables in the objective function are both non-negative, then the objective function has only a minimum value (provided that \( x \geq 0 \) and \( y \geq 0 \) are two of the constraints.)
- If the feasible region is the empty set, the linear programming problem has no solution.

**Method of Corners for Bounded Solution Set**

- Graph the feasible region.
- Find the coordinates of all corner points.
- Make a table and evaluate the objective function at all corner points to see which yields the optimal solution. (Note: If the objective function is optimized at two corner points, then the objective function is optimized at every point along the line segment connecting these two points as well. We say we have infinitely many solutions.)

- Please note that if we have an unbounded feasible region, depending on the coefficients in our objective function, a maximum or minimum value may not be possible.

4. **Maximize** \( P = 3x + 2y \)

**Subject to**

\[
\begin{align*}
0.2x + 0.1y &\leq 1 \\ 0.15x + 0.3y &\leq 1.5 \\ 10x + 10y &\leq 60 \\ x &\geq 0, y &\geq 0 \\
\end{align*}
\]

**Graph**

\( y = -x + 6 \), \( y = -2x + 10 \), \( y = -0.2x + 1 \)

**Optimal Solution**

- \( P_{\text{max}} = 16 \) at the point \( (4,2) \)

---

**Table:**

<table>
<thead>
<tr>
<th>CP</th>
<th>( P = 3x + 2y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0</td>
</tr>
<tr>
<td>(0,5)</td>
<td>10</td>
</tr>
<tr>
<td>(2,4)</td>
<td>14</td>
</tr>
<tr>
<td>(4,2)</td>
<td>16</td>
</tr>
<tr>
<td>(5,0)</td>
<td>15</td>
</tr>
</tbody>
</table>

**A maximum value of 16 obtained at the point \((4,2)\)**
5. Maximization Problem:

\[ P = 2x + 3y \]
\[ 5x + 4y \geq 56 \]
\[ x + 2y \geq 22 \]
\[ x \geq 0, y \geq 0 \]

Constraints:
- \( x_{\min} = 0 \)
- \( y_{\min} = 0 \)

Feasible Region:
- \( y_1 = (-\frac{5}{4})x + 14 \)
- \( y_2 = (-\frac{7}{2})x + 11 \)

Since we are trying to maximize the objective function and the feasible region is unbounded, we can conclude that the problem has **No Solution**.
6. Minimize \( C = 2x + 4y \)

Subject to:
- \( 0.1x + 0.1y \geq 1 \)
- \( x + 2y \geq 14 \)
- \( 5x + 7y \leq 70 \)
- \( y \geq 0 \)
- \( x \geq 0 \)

\( x_{\text{min}} = 0 \), \( y_{\text{min}} = 0 \)

\( y_1 = -x + 10 \)
\( y_2 = (-1/2)x + 7 \)
\( y_3 = (-5/7)x + 10 \)

Feasible Region

A minimum value of 28 occurs at the points \((6, 4)\) and \((14, 0)\) and at every point on the line segment connecting the two points.
7. A serving of fruit salad has \(3\) grams of walnuts, \(3\) grams of fiber, and sells for \$4. A serving of vegetable salad has \(6\) grams of walnuts, \(2\) grams of fiber, and sells for \$2.

(a) How many servings of each type of salad would maximize revenue if you have \(24\) grams of walnuts and \(12\) grams of fiber available?
(b) Are there any leftover resources? Be specific.

\[
x = \text{# of servings of fruit salad they make \& sell}
\]

\[
y = \text{# of servings of vegetable salad}
\]

Maximize \(R = 4x + 2y\)

Subject to:

\[
3x + 6y \leq 24 \quad \text{walnuts}
\]

\[
3x + 2y \leq 12 \quad \text{fiber}
\]

\(x \geq 0, y \geq 0\)

Graph \(y = -\frac{1}{2}x + 4\)

Graph \(y = -\frac{3}{2}x + 6\)

Intersection of the two lines:

\[
-\frac{1}{2}x + y = -\frac{3}{2}x + 6
\]

\[
+\frac{3}{2}x - y = \frac{3}{2}x - 4
\]

\[x = 2\]

\[y = \frac{1}{2}(2) + 4 = 3\]

\((2, 3)\)

A maximum revenue of \$16 is obtained when they make 4 servings of fruit salad and 0 servings of vegetable salad.

\[4 \times 4 + 2 \times 0 = 16\]

\[4 \times 0 + 2 \times 4 = 8\]

\[2 \times 3 + 3 \times 4 = 14\]

\[1 \times 4 + 0 \times 2 = 16\]

\[b) \quad \text{Walnuts:} \quad 3x + 6y \leq 24 \quad 24 \text{ grams of walnuts avail.}
\]

\[
3(4) + 6(0) = 12 \text{ grams of walnuts used}
\]

\[24 - 12 = 12 \text{ grams of leftover walnuts}\]

\[\text{Fiber:} \quad 3x + 2y \leq 12 \quad 12 \text{ grams of fiber available}
\]

\[
3(4) + 2(0) = 12 \text{ grams of fiber used}
\]

\[12 - 12 = 0 \quad \text{No leftover fiber}\]