Section 6.1

- We can either use roster notation or set-builder notation to represent a set.
- Two sets \( A \) and \( B \) are equal, written \( A = B \), if and only if they have exactly the same elements.
- If every element of a set \( A \) is also an element of a set \( B \), then we say that \( A \) is a subset of \( B \) and write \( A \subseteq B \).
- If \( A \) and \( B \) are sets such that \( A \subseteq B \) but \( A \neq B \), then we say \( A \) is a proper subset of \( B \) written \( A \subset B \).
- The set that contains no elements is called the empty set and is denoted by \( \emptyset \). It is a subset of all sets.
- The universal set is the set of all elements of interest in a particular problem.
- We use Venn Diagrams to visually represent sets. The universal set \( U \) is represented by a rectangle and subsets of \( U \) are represented by circles inside of the rectangle.
- The union of \( A \) and \( B \), written \( A \cup B \) is the set of all elements that belong to \( A \) or \( B \).
- The set of elements in common with the sets \( A \) and \( B \), written \( A \cap B \), is called the intersection of \( A \) and \( B \).
- The complement of \( A \), denoted \( A^c \) is the set of all elements in \( U \) that are not in \( A \).
- Two sets \( A \) and \( B \) are disjoint if \( A \cap B = \emptyset \).
- De Morgan's Laws: Let \( A \) and \( B \) be sets. Then \((A \cup B)^c = A^c \cap B^c\) and \((A \cap B)^c = A^c \cup B^c\).

1. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), \( A = \{1, 3, 5, 7, 9\} \), \( B = \{2, 4, 6, 8, 10\} \), \( C = \{1, 5, 8, 9\} \), \( D = \{8, 10\} \). Find the following:
   (a) \( A \cup C \)
   (b) \( B \cap D \)
   (c) \( C^c \)
   (d) \( A \cup (B \cap D) \)
   (e) \( C \cap (A \cup D)^c \)

2. Using the following sets, determine whether each statement is True or False.
   \( U = \{1, 2, 3, 4, 5, 9, 12, 15, 18\} \)
   \( A = \{2, 4, 9, 15\} \), \( B = \{1, 3, 15, 18\} \), \( C = \{5, 9, 15\} \)
   (a) \( \{2, 15\} \in A \)
   (b) \( (B \cup C)^c \cap A^c = \emptyset \)
   (c) \( A \) has 16 subsets
   (d) \( \emptyset \subset B \)
   (e) \( \emptyset \cup C = \{5, 9, 15\} \)
   (f) \( A \) and \( B \) are disjoint sets.
3. List all subsets of the set \( A \) where \( A = \{ x \mid x \text{ is an integer between } 3 \text{ and } 5 \text{ inclusive} \} \).

4. For each part, draw a Venn Diagram with 3 subsets, \( A \), \( B \), and \( C \), and shade the region that represents each set.
   (a) \( B \cap A^c \cap C^c \)  
   (b) \( C \cap (A \cup B) \)  
   (c) \( A \cup (B \cap C)^c \)

Section 6.2

- \( n(A) \) represents the number of elements in a set.
- \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)
- We can label each region in a Venn Diagram with the number of elements in it to sort out the given information.

5. If \( n(A) = 15 \), \( n(B) = 23 \), and \( n(A \cap B) = 5 \), what is \( n(A \cup B) \)?

6. A Universal Set consists of 3 subsets. We are given the following information:
   - \( n(A \cap B^c \cap C^c) = 10 \)
   - \( n(A) = 25 \)
   - \( n(B \cap C) = 10 \)
   - \( n(C) = 30 \)
   - \( n(B^c) = 35 \)
   - \( n(A \cap C \cap B^c) = 9 \)
   - \( n(A \cap B \cap C) = 2 \)
   - \( n(C^c) = 22 \).

Fill out the Venn Diagram with this information and use it to find \( n(B \cup C) \).

7. A survey was conducted of 100 student’s dining preferences. They were all asked their preference on eating at Chipotle, Fitzwilly’s, and Potbelly’s. The following information was determined:
   - 20 students like only Fitzwilly’s.
   - 30 students like exactly two of the restaurants.
   - 50 students like Potbelly’s
   - 5 students like all three restaurants.
   - 15 students like Chipotle and Fitzwilly’s.
   - 8 students like only Fitzwilly’s and Potbelly’s
   - 60 students do not like Chipotle.

Fill out the Venn Diagram with this information and use it to determine how many students do not like Fitzwilly’s.
Section 6.3

- Multiplication Principle: The total number of ways to perform a large task is the product of the number of ways to perform each subtask.

8. There are six issues on the county election ballot. If a voter can mark YES, NO, or choose not to vote on each issue, how many ways can the ballot be marked?

9. Five students are sitting in the waiting room to see an advisor. In how many ways can these students be directed to the departments' ten advisors if each advisor handles no more than one student?

10. A group of 8 people are going on a trip in an 8 seater van that was rented. If Jack and Jill were the only people that signed the papers to drive the van, how many different seating arrangements are possible?

11. How many three digit numbers can be formed from the digits 1, 3, 4, 5, 6, 9 if the numbers formed must all be even and no digit can be repeated?

12. You have 5 different thin bottles and 5 different wide bottles that you are going to arrange in a row on a shelf. How many different arrangements of these bottles are possible if the thin bottles and the wide bottles alternate?

13. Four Gamecoks, five Aggies, and three Wildcats travel to the SEC Basketball Tournament. If they all want to sit in the same row together, how many ways can this happen if each school wants to sit as a group?

14. Six letter code words are formed from the first ten letters of the English alphabet. If repetitions of the letters is not allowed, how many code words consist of exactly one vowel if the first letter must be B or C?