

Math 166 - Spring 2008

Week-in-Review #11

Final Exam Review

courtesy: Kendra Kilmer

(covering A.1 & A.2 & Chap 2,5,6,7,8,&9 (excluding Sec 5.4&9.3))

(Please note that this review is not all inclusive)

- How many 3 digit numbers can be formed from the numbers 0, 1, 2, 5, 7, 8, 9 if no digit can be repeated, the first digit must be nonzero, and each number formed must be even?
- A \$10,000 1-year term life insurance policy costs \$150. If the probability that a particular person dies in the next year is 0.002, find the company's expected net gain.
- Assume that a small town has only three industries: auto, energy, and transportation. Each of these industries acts as a supplier and a user of their products. The input-output matrix ( $A$ ) for this economy is given by

$$\begin{array}{r}
 \text{auto} \\
 \text{energy} \\
 \text{transportation}
 \end{array}
 \begin{array}{c}
 \text{auto} \quad \text{energy} \quad \text{transportation} \\
 \left[ \begin{array}{ccc}
 0.2 & 0.4 & 0.1 \\
 0.1 & 0.2 & 0.2 \\
 0.2 & 0.2 & 0.1
 \end{array} \right]
 \end{array}$$

Find the value of the goods consumed in the internal process of production in order to satisfy an external demand for automobiles of \$474 million, external demand for energy of \$948 million, and an external demand for transportation of \$474 million.

- Determine whether each of the following is a proposition:
  - Take me to school.
  - The school is on the right-hand side of the road.
  - $3x + 5 = 20$
- Given the payoff matrix below:
 
$$\begin{bmatrix} -1 & 3 & -2 \\ 5 & -4 & 2 \end{bmatrix}$$
 Find the expected payoff of the game if the row player selects row 1  $1/8$  of the time and the column player selects column 1  $1/5$  of the time and column 2  $2/5$  of the time.
- An experiment consists of observing whether cars are kept in the driveway or in the garage. It was found that 75% of cars are kept in the driveway, and the remaining cars are kept in the garage. Of those cars kept in the driveway, 90% were worth less than \$15,000. Of those kept in the garage, 80% were worth \$15,000 or more. If a car worth less than \$15,000 is randomly selected, what is the probability that it is in the driveway?
- Solve the following system of linear equation using inverses:

$$3x + 7y - 15z = 9$$

$$z = 5x - 9y + 7$$

$$2x - 5y = z + 9$$

- A girl scout convention is being held in College Station. Troop 647 is bringing 7 girls, Troop 916 is bringing 5 girls, and Troop 525 is bringing 3 girls. In how many ways can these girls sit in a row if each troop wants to sit together?

- A research study was done on consumers' behavior regarding their toilet paper preferences. It was found that each month, of the consumers using Brand A, 70% will continue to use Brand A whereas 25% will switch to Brand B, and 5% will switch to Brand C. Of the consumers using Brand B, 80% will continue to use Brand B whereas 5% will switch to Brand A and 15% will switch to Brand C. Of the consumers using Brand C, 95% will continue to use Brand C whereas 1% will switch to Brand A and 4% will switch to Brand B. At the beginning of January, it is known that 33% of the consumers are using Brand A, 62% of the consumers are using Brand B, and 5% of the consumers are using Brand C. What percent of the consumers will be using Brand A at the beginning of June? What percent of the consumers will be using Brand A in the long run?

- Pivot the given matrix about the element  $a_{22}$ :

$$\left[ \begin{array}{ccc|c}
 1 & -2 & 5 & 9 \\
 0 & 4 & -8 & 16 \\
 0 & 5 & 3 & 9
 \end{array} \right]$$

- Solve the following matrix equation for  $a$ ,  $b$ , and  $c$ :

$$\begin{bmatrix} 7 & 5 & 6b \\ 9-c & 4 & 2 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} a & 9 & 2 \\ 10 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 81 & 96 & 108 \\ 100 & 84 & 75 \end{bmatrix}$$

- Given  $U = \{1, 2, 3, 4, 5, 6\}$ ,  $A = \{1, 5\}$ ,  $B = \{1, 3, 5, 6\}$ , and  $C = \{2, 4, 6\}$ , determine whether each statement is true or false.
  - $\{1, 3\} \in B$
  - $U$  has 64 proper subsets.
  - $A$  and  $B$  are disjoint sets.
  - $(A \cap B)^c \cup C = \{2, 4, 6\}$ .
  - $2 \in (A \cup B)^c$

- What is the effective rate of interest of an account if the interest is compounded monthly and the investment will double in 20 years?

- A group of 488 Aggies are going on a road trip to the bowl game. They are trying to decide how many cars, vans, and buses to take for the trip. They has determined that each car will seat 6 people and the expenses involved for each car come out to be \$100. Each van holds 14 people and costs \$195 per van. Each bus holds 40 people and costs \$475. If they have decided that they want to take twice as many buses as vans, how many vehicles of each type should they take if the budget is \$6,025 and they want to exactly use all of their budget and have no empty seats?

- Any purchases a \$250,000 house. She makes a 5% down payment and then finances the remaining balance with a 30 year mortgage that charges a 6.25% annual interest rate compounded monthly on the unpaid balance.
  - What is her required monthly payment?
  - Assuming she makes exactly the required payment each month, create an amortization schedule detailing the 1st, 2nd, 241st, and 242nd payments.

- Consider the economy of a small village consisting of two industries, farming and weaving. The farmers produce food for themselves and the weavers, along with extra food for the remaining people. The weavers produce cloth for themselves and

the farmers, along with extra cloth for the remaining people. The villagers have found that to produce \$1.00 of food, \$0.40 of food and \$0.10 worth of cloth is needed locally to feed and clothe the farmers. To produce \$1.00 of cloth, \$0.30 of food and \$0.20 of cloth is needed locally to feed and clothe the weavers. The additional food and cloth produced is then available to export to the city. If the city demands \$7,200 worth of food and \$2,700 worth of cloth each month, how much food and cloth should be produced by the village to meet its own needs and to supply the city?

17. A survey of 550 freshmen revealed that 300 own a car, 200 own a computer and 100 own neither a car nor a computer. If a student is randomly selected from this group what is the probability that they own a car but not a computer?

18. An experiment consists of randomly selecting one card out of a standard deck of 52 cards. Let  $E$  be the event that a red card is drawn and let  $F$  be the event that a club is drawn. Are the events  $E$  and  $F$  mutually exclusive events? Are the events  $E$  and  $F$  independent events?

19. Katie bought a house in 2000 for \$300,000. She financed it with a 30 year mortgage at an annual interest rate of 7.25% compounded monthly on the unpaid balance.

- (a) What are her current monthly payments?
- (b) After making six years of payments she decides to refinance her home in 2006 with a 30 year mortgage that has an annual interest rate of 5.25% compounded monthly on the unpaid balance. What are her new monthly payments?
- (c) How much total interest will she end up paying for this house?

20. The independent probabilities that Jenny, Karen, and Lindsay will attend class are 0.35, 0.95, and 0.76, respectively. What is the probability that exactly two of the girls will attend class?

21. For each of the following payoff matrices (i) Determine the maximin and minimax strategies. (ii) Is the game strictly determined? If so, what is the saddle point/value of the game? Who does the game favor?

- (a)  $\begin{bmatrix} -1 & 3 & -2 \\ 5 & -4 & -3 \end{bmatrix}$
- (b)  $\begin{bmatrix} 4 & -3 & 5 \\ 2 & 7 & -1 \\ -3 & 2 & 1 \end{bmatrix}$

22. A jar contains 10 pennies and 5 dimes. Three coins are randomly selected from the jar. Let  $X$  represent the value of the three coins (i.e. how much money you have). Find the probability distribution of  $X$ .

23. How many days will it take for a sum of \$2,000 to earn \$50 interest if it is deposited in an account earning simple interest at the rate of 8% per year?

24. Classify each of the random variables as finite discrete, infinite discrete, or continuous:

- (a) Let  $X$  represent the weight of a newborn kitten in ounces.

(b) Cough drops are randomly selected with replacement from a bag containing 30 cherry, 20 menthol, and 35 lemon. Let  $Y$  represent the number of cough drops drawn until a cherry cough drop is selected.

(c) Jane serves the volleyball 100 times. Let  $Z$  represent the number of times she serves an ace.

25. Two marbles are randomly selected from a bin containing 8 red and 5 green marbles. If this experiment is repeated 10 times and the marbles are replaced each time, what is the probability of getting 2 red marbles at least 3 times?

26. Sarah is trying to arrange 5 identical blue candle sticks, 7 identical red candle sticks, and 3 identical green candle sticks on her shelf. How many distinguishable arrangements are possible?

27. Given the following matrices with the indicated dimensions, which of the following are valid matrix operations?

$$A_{2 \times 4}, B_{3 \times 1}, C_{4 \times 3}, D_{3 \times 2}$$

- (a)  $AC - 6D^T$
- (b)  $5D^T B$
- (c)  $2CB$
- (d)  $A^T - CD$

28. An experiment consists of rolling a fair six-sided die 1000 times. Find the probability of rolling a 5 at most 170 times. Use the appropriate normal distribution to approximate the binomial probability.

29. Valerie decides to put \$30 into an account every week. If the account earns 3.45%/year compounded weekly, how much will she have in the account 15 years from now?

30. Given the following probability distribution:

$x$	-15	-5	20	30
$P(X = x)$	0.056	0.72	0.19	0.034

Find  $E(X)$ ,  $Var(X)$ ,  $\sigma$ , median, and mode.

31. Given  $S = \{2, 4, 6, 8, 10, 12\}$ ,  $A = \{2, 6, 12\}$ ,  $B = \{8, 10, 12\}$ ,  $C = \{2, 4, 6, 8, 10\}$ , find the following:

- (a)  $A^c \cap B$
- (b)  $A \cup (B \cap C)^c$
- (c)  $B \cap (A \cap C)$

32. In College Station, 85% of the population is currently an Aggie fan. Each year 30% of the non-Aggie fans become Aggie fans and 1% of the Aggie fans become non Aggie fans. If this trend continues, what percent of the population will be an Aggie fan 5 years from now?

33. If  $n(A \cup B) = 80$ ,  $n(A) = 30$ , and  $n(B) = 75$ , what is  $n(A \cap B^c)$ ?

34. Jenny and Lisa have recently acquired some leftover inventory from a small online baby supply store. Jenny has acquired 20 pacifiers, 18 bottles, and 9 cloth diapers. Lisa has acquired 15 pacifiers, 7 bottles, and 12 cloth diapers.

- (a) Organize the above information into a matrix and label it  $A$ .

(b) The pacifiers will sell for \$2 each, the bottles will sell for \$4 each, and the cloth diapers will sell for \$12 each. Write a matrix  $B$  summarizing this data.

(c) Write a matrix product involving the matrices  $A$  and  $B$  that will produce a new matrix giving the total amount of money each girl will make if they sell all of their inventory. Compute the desired product and interpret the result.

35. Let  $X$  be a normal random variable with a mean of 0 and a standard deviation of 8. Find the value of  $b$  if  $P(-b < X < b) = 0.9826$ .

36. Given the payoff matrix below determine the optimal mixed strategy for the row and column player and then compute the expected payoff of the game. Who does the game favor?

$$\begin{bmatrix} -5 & -2 \\ 6 & -4 \end{bmatrix}$$

37. Let  $p$  and  $q$  denote the propositions

$p$ : It will be hot this summer.

$q$ : I will spend the summer on the beach.

Express the following compound propositions in words:

(a)  $\sim p \wedge q$

(b)  $p \vee q$

(c)  $p \vee \sim q$

38. A True/False Exam consists of 15 questions. In how many ways can the exam be answered so that the student answers at least 10 questions correctly?

39. An experiment consists of rolling a fair six-sided die and flipping a fair coin.

(a) Find the Sample Space associated with this experiment.

(b) Find the event,  $E$ , that an even number is rolled.

(c) Find the  $P(E)$ .

40. Gabe has \$2,500 in credit card debt on a card that charges 19.6% per year compounded monthly on the unpaid balance. If he pays the minimum payment of \$50 each month, and he doesn't use it to make any more purchases, how long will it take Gabe to pay off his debt?

41. An experiment consists of rolling a pair of fair six-sided dice. What is the probability that the sum is at least 9 or exactly one 5 is rolled?

42. An experiment consists of randomly selecting 1 marble from a bowl containing 5 red, 3 blue, and 10 white marbles. What is the probability that the marble selected is not white?

43. Solve the systems of linear equation using the Gauss-Jordan elimination method.

(a)

$$-12x - 42y - 48z = -114$$

$$2x + 7y + 8z = 19$$

$$3x + 2y + 10z = 9$$

(b)

$$2x + 3y - 2z = 10$$

$$x + 3y - z = -3$$

$$3x + 4y - z = 7$$

(c)

$$3x - 9y + 6z = 12$$

$$x - 3y + 2z = -4$$

$$2x - 6y + 4z = 8$$

44. If  $E$  and  $F$  are mutually exclusive events,  $P(E) = 0.5$ , and  $P(F) = 0.3$ , find  $P((E \cap F^c) \cup (E^c \cap F))$ .

45. A crate contains 90 strawberries of which 10 are rotten. If a customer randomly selects 15 strawberries, what is the probability that at least 3 of the strawberries are rotten?

46. A fruit stand has 30 apples, 14 peaches, 6 plums, and 15 pears. If you randomly select 8 pieces of fruit, how many ways can you select exactly 4 plums or exactly 3 pears?

47. In a group of 10 people, what is the probability that at least 2 of them were born in the same month?

48. An experiment consists of first performing a task in which  $A$ ,  $B$ , or  $C$  can occur. Then, a second task is performed in which  $D$  or  $E$  can occur. The following probabilities are known:  $P(A) = 0.3$ ,  $P(C) = 0.5$ ,  $P(D|A) = 0.25$ ,  $P(E|B) = 0.7$ ,  $P(D|C) = 0.64$ . Find the following:

(a)  $P(E)$

(b)  $P(B|E)$

(c)  $P(A \cup D)$

49. The scores on an English exam were normally distributed with a mean of 65 and a standard deviation of 5. If 10% of the class received an  $A$  on the exam and 15% of the class received a  $B$  on the exam, what was the cutoff for a  $B$ ?

50. A club consists of 35 finance majors, 30 accounting majors, and 40 info majors. This club must elect a president and vice-president from the finance majors and then elect 2 representatives from each of the two remaining majors. How many ways can this be done?

51. The following is the transition matrix for a Markov process that has 3 states:  $A$ ,  $B$ , and  $C$ . (Note: the rows and columns are represented in the matrix in the order listed above.)

$$T = \begin{bmatrix} 0.1 & 0.5 & 0.3 \\ 0.9 & 0.2 & 0.65 \\ 0 & 0.3 & 0.05 \end{bmatrix}$$

(a) Is the above matrix stochastic?

(b) Is the above matrix regular?

(c) What does the entry  $t_{32}$  represent?

(d) What does the entry  $t_{13}$  represent?

52. A game consists of rolling a pair of fair six-sided dice. If the sum of the numbers is less than 5, the player wins \$5. If the sum of the numbers is greater than 9, the player wins \$4. Otherwise, the player loses \$ $A$ . Find the value of  $A$  that makes this game fair.

53. It is known that 62% of Aggies purchased an all-sports pass. If 50 Aggies are randomly selected, what is the probability that at most 29 of them own a sports pass?

54. Shade the following regions on a Venn Diagram:

(a)  $A \cap (B \cup C)$

(b)  $B \cap C^c$

(c)  $(A \cap B) \cup (B^c \cap C)$

55. It is known that the length of newborns is normal distributed with a mean of 21 inches and standard deviation of 3 inches. What is the probability that a randomly selected newborn has a length between 18 and 24 inches?

56. A game consists of two players' simultaneously showing one, two, or three fingers. Carl and Rico decide to play this game. If the sum of the numbers showing is less than four Carl pays Rico \$3. If the sum of the numbers showing is greater than four, Rico pays Carl \$2. Otherwise, Carl pays Rico \$1.

(a) Find the payoff matrix for this two-player zero-sum game.

(b) Find the maximin and minimax strategies for Rico and Carl. Is this game strictly determined? If so, what is the value of the game? Who does this game favor?

57. Danny put a \$5,500 down payment on a car. He financed the remaining balance with a 60 month loan at a 5.5% annual interest rate compounded monthly on the unpaid balance. If the monthly car payments are \$450, what is the cash price of the car?

58. Determine the value of  $b$  for which the system of linear equations

$$4x - y = 6$$

$$8x + by = 10$$

has no solution.

59. A survey was done of 700 students on their preference of fruits. The following information was found:

- 275 students like bananas and strawberries.
- 250 students like all three fruits.
- 400 students like grapes.
- 50 students like only bananas and grapes.
- 20 students like only grapes.
- 205 students do not like bananas.
- 60 students like only strawberries.

How many students like exactly one of these kinds of fruit?

60. Construct a truth table for each compound proposition.

(a)  $\sim p \vee (p \wedge q)$

(b)  $(p \vee q) \wedge (\sim p \wedge r)$