

Math 166 - Spring 2008
Week-in-Review #3
Exam 1 Review
courtesy: Kendra Kilmer
(covering Sections 2.1-2.7, A.1-A.2, 6.1-6.2)
(Please note that this review is not all inclusive)

1. Determine whether each of the following is proposition:

- (a) Five is an even number.
- (b) There are 150 named streets in College Station.
- (c) Turn left at the second light.
- (d) $2x - 7 \geq 15$

2. Let p and q denote the propositions

p : The number of faculty members at Texas A&M increased over the past year.

q : The average class size at Texas A&M decreased over the past year.

Express the following compound propositions in words.

(a) $p \wedge \sim q$

(b) $\sim p \vee q$

(c) $\sim p \vee \sim q$

3. Let p and q denote the propositions

p : John went to the basketball game.

q : Crystal went to the basketball game.

Express each of the following statements symbolically.

(a) Crystal did not go to the basketball game but John did.

(b) Crystal went to the basketball game or John did not go to the basketball game.

(c) Neither John nor Crystal went to the basketball game.

4. Construct a truth table for each compound proposition.

(a) $(p \wedge q) \vee \sim p$

(b) $r \wedge \sim (p \vee q)$

5. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{1, 5, 8, 9\}$, $D = \{8, 10\}$. Find the following:

- (a) $A \cup C$
- (b) $B \cap D$
- (c) C^c
- (d) $C \cap (A \cup D)^c$

6. Using the following sets, determine whether each statement is True or False.

$$U = \{1, 2, 3, 4, 5, 9, 12, 15, 18\}$$

$$A = \{2, 4, 9, 15\}, B = \{1, 3, 15, 18\}, C = \{5, 9, 15\}$$

- (a) $\{2, 15\} \in A$
- (b) $(B \cup C)^c \cap A^c = \emptyset$
- (c) A has 16 subsets
- (d) $\emptyset \subset B$
- (e) $\emptyset \cup C = \{5, 9, 15\}$
- (f) A and B are disjoint sets.

7. For each part, draw a Venn Diagram with 3 subsets, A , B , and C , and shade the region that represents each set.

- (a) $B \cap A^c \cap C^c$
- (b) $C \cap (A \cup B)$
- (c) $A \cup (B \cap C)^c$

8. A survey was conducted of 100 student's dining preferences. They were all asked their preference on eating at Chipotle, Fitzwilly's, and Potbelly's. The following information was determined:

- 20 students like only Fitzwilly's.
- 30 students like exactly two of the restaurants.
- 50 students like Potbelly's
- 5 students like all three restaurants.
- 15 students like Chipotle and Fitzwilly's.
- 8 students like only Fitzwilly's and Potbelly's
- 60 students do not like Chipotle.

Fill out the Venn Diagram with this information and use it to determine how many students do not like Fitzwilly's.

9. Transportation Services is trying to decide on the width of the parking spaces in a newly created parking lot. To do so they have been doing a study on the size of cars that are on campus. During their study, they discovered that there are a total of 12,000 cars on campus that can be classified into three categories: compact cars, medium-sized cars, and trucks. The number of trucks is the same as the number of compact and medium-sized cars combined. There are half as many compact cars as medium-sized cars and trucks combined. Determine the number of each type of vehicle on campus during their study.

10. Solve the following system of linear equations by doing Gauss-Jordan Elimination by hand.

$$2x - 7y = 9$$

$$y = -3x + 5$$

11. Each of the following augmented matrices represents a system of linear equations. Determine whether or not the augmented matrix is in row-reduced form. If it IS in row-reduced form give the solution to the system of linear equations. If it IS NOT in row-reduced form, perform the next row operation that would need to be performed to get the augmented matrix in row-reduced form.

(a)
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

(b)
$$\left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & -7 \\ 0 & 0 & 15 \end{array} \right]$$

(c)
$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

12. Given the following matrices with the indicated dimensions, which of the following are valid matrix operations?

$$A_{4 \times 3}, B_{3 \times 2}, C_{4 \times 2}, D_{3 \times 4}$$

(a) $3AD$

(b) $A + 5D^T - B$

(c) $BC^T - 5D$

(d) $AB + 2D$

13. Solve the given matrix equation for a , b , and c .

$$\begin{bmatrix} 5 & a & -7 \\ 3 & -2 & 5 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 9 \\ -1 & 1 \\ 3 & b \end{bmatrix} - \begin{bmatrix} 4 & 1 & -3 \\ -5 & 2 & 7 \end{bmatrix}^T = \begin{bmatrix} -52 & 5 \\ 19 & 78 \\ c & 9 \end{bmatrix}$$

What is $2a + b - c$?

14. Consider a simple economy consisting of two sectors: goods and services. The production of 1 unit of goods requires the consumption of 0.1 unit of goods and 0.2 unit of services. The production of 1 unit of services requires the consumption of 0.3 unit of goods and 0.25 unit of services. Find the level of production for each sector in order to satisfy the demand for \$100 million worth of goods and \$400 million worth of services. How much of each sector is consumed internally while satisfying this demand?