Math 166 - Spring 2008
Week-in-Review #7
courtesy: Kendra Kilmer
(covering Sections 8.1-8.3)

Section 8.1

- A **random variable** is a rule that assigns a number to each outcome of an experiment.
- A **finite discrete** random variable is one which can only take on a limited number of values that can be listed.
- An **infinite discrete** random variable is one which can take on an unlimited number of values that can be listed in some sort of sequence.
- A random variable is **continuous** if the values it may assume comprise an interval of real numbers.
- We use **histograms** to visually represent probability distributions of random variables.

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Sections 8.2 and 8.3

- For a random variable X that takes on the values $x_1, x_2, \ldots x_n$ with associated probabilities p_1, $p_1, \ldots p_n$ The **expected value** of the random variable X is defined by $E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$
- A **fair game** is one in which the expected value of both players' net winnings is zero. (i.e. $E(X) = 0$)
- If $P(E)$ is the probability of an event occuring then $\frac{P(E)}{P(E^c)}$ is the **odds for the event occuring** and $\frac{P(E^C)}{P(E)}$ is the **odds against the event occuring**.
- If the odds of an event occuring are a to b, then the probability that the event will occur is $P(E) = \frac{a}{a+b}$.
- The **median** is the middle value when the data points are listed in increasing or decreasing order if there are an odd number of values. The median is the average of the middle two values if there are an even number of values.
- The **mode** is the value that occurs most frequently.
- The **variance**, Var(X), is a measure of how spread out the distribution is away from the mean. The **standard deviation**, σ, (also a measure of spread) is the square root of the variance.
- You can use 1-Var Stats to compute the mean, median, standard deviation, and variance. Make sure that you enter 1-Var Stats L_1, L_2 on the home screen if you enter in a probability distribution into L_1 and L_2.
- If a random variable has a mean of μ and a standard deviation of σ then **Chebychev's Inequality** allows you to estimate the probablity that a randomly selected outcome will be within k standard deviations of the mean. It states:

$$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

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1. Classify each random variable as finite discrete, infinite discrete, or continuous AND describe/list the possible values of the random variable.

(a) A bag contains 10 purple skittles, 15 red skittles, and 20 green skittles. An experiment consists of randomly selecting skittles with replacement out of the bag until a purple skittle is selected. Let X represent the number of skittles drawn.

1, 2, 3, ...
$\boxed{\text{infinite discrete}}$

(b) An experiment consists of randomly selecting a sample of twenty oranges out of a crate containing 100 oranges of which 10 are rotten. Let Y represent the number of rotten oranges selected.

0, 1, 2..., 10
$\boxed{\text{finite discrete}}$

(c) An experiment consists of randomly selecting a sample of twenty oranges out of a crate containing 100 oranges of which 10 are rotten. Let Z represent the amount of time it takes you to select the sample of 20 oranges.

$z > 0$ $\boxed{\text{continuous}}$

(d) An experiment consists of rolling a fair six-sided die five times. Let W represent the number of times the die lands on 2.

0, 1, 2, 3, 4, 5
$\boxed{\text{finite discrete}}$

(e) An experiment consists of randomly selecting 5 cards (with replacement) out of a standard deck of 52 cards. Let M represent the number of spades drawn.

0, 1, 2, 3, 4, 5
$\boxed{\text{finite discrete}}$

(f) An experiment consists of randomly selecting cards (without replacement) out of standard deck of 52 cards. Let N represent the number of cards drawn until a king is drawn.

1, 2, 3, ..., 49 $\boxed{\text{finite discrete}}$

(g) An experiment consists of measuring the amount of water in an aquarium that can hold up to 30 gallons of water. Let P represent the actual amount of water in the aquarium (measured in gallons).

0 1 30

$0 \leq p \leq 30$ $\boxed{\text{continuous}}$

(h) An experiment consists of randomly selecting a banana out of the produce department and weighing it in ounces. Let Q represent the weight of the banana.

$q > 0$ $\boxed{\text{continuous}}$

(i) An experiment consists of polling students on the exact amount of sleep they get in a 24 hour day. Let R be the amout of sleep in hours.

$0 \leq r \leq 24$ $\boxed{\text{continuous}}$

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2. An experiment consists of randomly selecting a sample of three coins out of a bowl containing 8 nickels and 12 dimes. Let X represent the monetary value of the coins in the sample. Find the $E(X)$.

x	$P(X=x)$
.15	$\frac{C(8,3)\cdot C(12,0)}{C(20,3)} = \frac{56}{1140}$
.20	$\frac{C(8,2)\cdot C(12,1)}{C(20,3)} = \frac{336}{1140}$
.25	$\frac{C(8,1)\cdot C(12,2)}{C(20,3)} = \frac{528}{1140}$
.30	$\frac{C(8,0)\cdot C(12,3)}{C(20,3)} = \frac{220}{1140}$

8N
12D
select 3

3N 0D 15¢
2N 1D 20¢
1N 2D 25¢
0N 3D 30¢

$$E(X) = .15\left(\frac{56}{1140}\right) + .2\left(\frac{336}{1140}\right) + .25\left(\frac{528}{1140}\right) + .3\left(\frac{220}{1140}\right)$$
$$= \boxed{\$.24}$$

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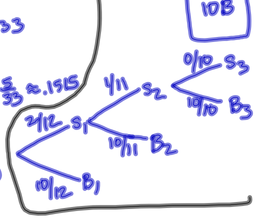
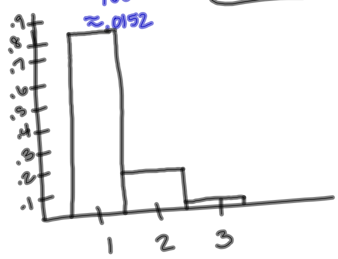
3. A bowl contains two snickers and ten butterfingers. An experiment consists of randomly drawing a piece of candy out of the bowl (without replacement) until a butterfinger is drawn. Let X represent the number of pieces of candy drawn until a butterfinger is drawn. Find the probability distribution of X and then draw the histogram for X.

x	$P(X=x)$
1	$10/12 \approx .8333$
2	$(2/12)(10/11) = \frac{5}{33} \approx .1515$
3	$(2/12)(1/11)(10/10) = 1/66 \approx .0152$

2S
10B





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4. In a lottery, 5000 tickets are sold for \$2 each. One grand prize of \$10,000 and 100 consolation prizes of \$100 are to be awarded. What are the expected net earnings of a person who buys one ticket?

Let X represent the net earnings of a person who buys one ticket

x	$P(X=x)$
-2	4899/5000
9998	1/5000
98	100/5000

$$E(X) = -2(4899/5000) + 9998(1/5000) + 98(100/5000)$$
$$= \boxed{-\$2}$$

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5. Donna wants to purchase a 3-year term life insurance policy that will pay the beneficiary $50,000 in the event that Donna doesn't survive the next three years. Using life insurance tables, she determines that the probability that she will live another three years is 0.998. If the company charges $700 for the policy, what is the company's expected net gain?

Let X represent the company's net gain

x	$P(X=x)$
700	.998
$700-50000 = -49300$	$1-.998 = .002$

$E(X) = 700(.998) + (-49800)(.002)$

$= \boxed{\$600}$

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6. Jack and Jill decide to play a game. Jill rolls a pair of fair six-sided dice. If the sum of the numbers is less than 4, Jill pays Jack $1. If the sum of the number is greater than 9, Jack pays Jill $10. Otherwise, Jill pays Jack $*A*. Find the value of *A* that makes this game fair.

Let X represent Jills net winnings

x	$P(X=x)$
-1	$3/36$
10	$6/36$
$-A$	$27/36$

	1	2	3	4	5	6
1	X	X				
2	X					
3						
4						O
5					O	O
6				O	O	O

Fair $\Rightarrow E(X)=0$

$-1(3/36) + 10(6/36) + (-A)(27/36) = 0$

$\frac{19}{12} - \frac{3}{4}A = 0$

$\left(\frac{4}{3}\right)\frac{19}{12} = \frac{3}{4}A\left(\frac{4}{3}\right)$

$\boxed{\$2.11 \approx A}$

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7. Suppose you play a game in which you pay a certain amount to draw two cards out of a standard deck of 52. You receive twice of what you paid if both cards drawn are aces. You don't receive anything if the two cards drawn are from the same suit. For anything else, you receive $6. How much should be charged to make this a fair game?

Let X represent our net winnings

Let a represent the amount you pay to play the game

x	$P(X=x)$
$2a-a = a$	$\frac{C(4,2)}{C(52,2)} = \frac{6}{1326}$
$-a$	$\frac{4 \cdot C(13,2)}{C(52,2)} = \frac{312}{1326}$
$6-a$	$\frac{1008}{1326}$

Fair $\Rightarrow E(X)=0$

$a(6/1326) - a(312/1326) + (6-a)(1008/1326) = 0$

$-\frac{3}{13}a + \frac{1008}{221} - \frac{168}{221}a = 0$

$-\frac{219}{221}a + \frac{1008}{221} = 0$

$\left(\cancel{\frac{-221}{219}}\right)\cancel{\frac{-219}{221}}a = \frac{-1008}{221}\left(\frac{-221}{219}\right)$

$\boxed{a \approx \$4.60}$

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8. The odds of an event occurring are 3 to 8. What is the probability that the event will not occur?

$\frac{8}{3+8} = \boxed{\frac{8}{11}}$

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9. The weather forecaster predicts that the probability that it will rain on Friday is 0.38. What are the odds that it will NOT rain on Friday?

$P(E) = .38$

$\frac{P(E^c)}{P(E)} = \frac{1-.38}{.38} = \frac{.62}{.38} = \frac{31}{19}$

$\boxed{31 \text{ to } 19}$

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10. Find the odds of drawing an ace, from a standard deck of cards, on the second draw if we know that the first card drawn was a king.

$P(E) = \frac{4}{51}$

$\frac{P(E)}{P(E^c)} = \frac{4/51}{1-4/51} = \frac{4/51}{47/51} = \frac{4}{47}$

$\boxed{4 \text{ to } 47}$

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11. The table below gives the results of an experiment in which the number of green skittles in randomly selected bags was recorded.

Number of Green Skittles	20	21	22	23	24
Number of Bags	35	42	50	51	21

Total 199

Find the mean and standard deviation of the number of green skittles in a randomly selected bag.

X = # of green skittles

x	20	21	22	23	24
$P(X=x)$	35/199	42/199	50/199	51/199	21/199

← L_1 ← L_2

1-Var stats L_1, L_2

$\boxed{\bar{x} = 21.9045}$ ← mean

$\boxed{\sigma_x = 1.2585}$ ← Std Dev.

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12. An experiment consists of counting the number of prime factors in the factorization of two digit numbers. The results are given below:

~~Three~~ Two Digit Number	16	25	36	49	64	81
Number of Prime Factors	4	2	4	2	6	4

Based on this experiment, find the mean and standard deviation of the number of prime factors in a two digit number.

X = # of prime factors in a two digit number

L_1: 4, 2, 4, 2, 6, 4

1-Var Stats

$\bar{x} = 3.6667$ (mean)
$\sigma_x = 1.3744$ (std dev)

OR//

x	$P(X=x)$
2	2/6
4	3/6
6	1/6
↑ L_1	↑ L_2

1-Var stats L_1, L_2

$\bar{x} = 3.6667$ (mean)
$\sigma_x = 1.3744$ (std. dev)

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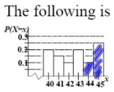
13. The following is a histogram for the random variable X.



(a) Find $E(X)$. $E(X) = 40(.2) + 41(.15) + 42(.1) + 43(.2) + 44(.1) + 45(.25) = \boxed{42.6}$

(b) What is the mode? $\boxed{45}$ (tallest bar/highest prob of occurring)

(c) What is the median? If I listed out 100 data values,

20 40's
15 41's
10 42's
20 43's
10 44's
25 45's

Avg of the 50th, 51st $\boxed{43}$

OR// when adding the cum. prob its the bar that puts you over .5 $\boxed{43}$

(d) What is the standard deviation? $\sigma_x = 1.8547$

(e) What is the variance? $(1.854723699)^2 = \boxed{3.44}$

(f) Find $P(X > 43)$ $.1 + .25 = \boxed{.35}$

L_1	L_2
40	.2
41	.15
42	.1
43	.2
44	.1
45	.25

1-Var stats L_1, L_2

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14. A Christmas tree light has an expected life of 205 hours and a standard deviation of 2 hours. Use Chebychev's Inequality to estimate the probability that one of these Christmas tree lights will last between 197 and 213 hours.

$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$

$\mu = 205$
$\sigma = 2$

$P(197 \le X \le 213) \ge 1 - \frac{1}{4^2} = 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$

k = # of times we add the std dev to the mean to get to the right bound

205 ($= \mu$), 207, 209, 211, 213

Here, k = 4

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15. A random variable X has a mean of 214 and a standard deviation of 8. Use Chebychev's Inequality to determine the value of c for which $P(214 - c \le X \le 214 + c) \ge 0.99$.

$\mu = 214$
$\sigma = 8$

$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2}$

$1 - \frac{1}{k^2} = .99$
$-1 \qquad -1$
$-\frac{1}{k^2} = -.01$
$\frac{1}{k^2} = .01$
$\frac{1}{k^2} = \frac{1}{100}$
$k^2 = 100$
$k = 10$

$c = 10(8) = \boxed{80}$

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