

Math 166 - Spring 2008  
Week-in-Review #8  
courtesy: Kendra Kilmer  
(covering Sections 8.4-8.6)

### Section 8.4

- A **binomial experiment** has the following properties:
  - The total number of trials is fixed in advance.
  - Each trial has two outcomes: “success” and “failure”.
  - The trials are independent of each other.
  - The probability of success is the same for each trial.
- The random variable  $X$  which represents the number of successes in a binomial experiment is known as a **Binomial Random Variable**.
- If we are dealing with a binomial random variable, we must determine the number of trials ( $n$ ), the probability of success in each trial ( $p$ ), and the number of successes desired ( $r$ ). We can then compute the probabilities by hand or on the calculator:
  - By Hand:  $P(X = r) = C(n, r)p^r(1 - p)^{(n-r)}$
  - On the Calculator (Hit 2nd VARS)
    - \* To calculate a single probability: binompdf(n,p,r)
    - \* To add up the probabilities for  $r$  from 0 to  $k$ : binomcdf(n,p,k)
- For a binomial random variable we can easily compute statistics using the following shortcuts:
  - $E(X) = np$
  - $Var(X) = np(1 - p)$
  - $\sigma = \sqrt{np(1 - p)}$

1. Is the following random variable binomial?

- An experiment consists of randomly selecting a sample of twenty oranges out of a crate containing 100 oranges of which 10 are rotten. Let  $Y$  represent the number of rotten oranges selected.
- An experiment consists of rolling a fair six-sided die five times. Let  $W$  represent the number of times the die lands on 2.
- An experiment consists of randomly selecting 5 cards (with replacement) out of a standard deck of 52 cards. Let  $M$  represent the number of spades drawn.
- An experiment consists of randomly selecting cards (without replacement) out of standard deck of 52 cards. Let  $N$  represent the number of cards drawn until a king is drawn.

2. A fair six-sided die is rolled 50 times.

- What is the probability of rolling a five 10 times?
- What is the probability of rolling an even number 20 times?
- What is the probability of rolling a 3 at most 10 times?
- What is the probability of rolling an odd number at least 35 times?
- How many times would you expect to roll an even number?
- What is the standard deviation of the number of times you roll an even number?

- On a given class day 15% of the students are not listening in their class. If 25 students are randomly selected,
  - What is the probability that exactly 15 of the students are listening?
  - What is the probability that at least 16 of the students are listening?
  - What is the probability that between 16 and 20 students, inclusive, are listening?
  - How many students would you expect to be listening in the group of 25?
  - What is the standard deviation of the number of students listening in this group of 25?
- Suppose that one-fifth of the restaurants in town are in violation of the health code. If an inspector randomly inspects seven of the restaurants,
  - What is the probability that the first four restaurants will pass the inspection and the remaining three will fail the inspection?
  - What is the probability that only four restaurants will pass the inspection?

### Section 8.5

- If  $X$  is a **continuous random variable**, a **probability density function** is defined to represent the probability distribution of  $X$ . The curve lies completely above the  $x$ -axis and the total area under this curve is 1.
- A normal random variable is defined by its mean ( $\mu$ ) and standard deviation ( $\sigma$ ). The probability density function associated with a normal random variable has its peak directly above the mean and is symmetric about a vertical line passing through the mean. The **standard normal variable**  $Z$  has a mean of 0 and a standard deviation of 1.
- To find probabilities associated with a Normal Random Variable, press 2nd VARS and then select option 2:normcdf. On your homescreen, enter normcdf(left bound, right bound, mean, standard deviation).
- If you are given a probability and asked to find a bound use 3:invNorm which is also found by pressing 2nd VARS. If you are trying to find  $a$ , then  $a = \text{invNorm}(\text{total area under the curve to the left of } a, \text{ mean, standard deviation})$ .

5. Let  $Z$  be the standard normal variable. Find the following:

- $P(Z \geq 1.5)$
- $P(Z < 2)$
- $P(-1.5 < Z \leq 2)$

6. Let  $X$  be a normal random variable with  $\mu = 80$  and  $\sigma = 5$ . Find the following:

- $P(X \leq 70)$
- $P(X > 75)$
- $P(45 \leq X \leq 90)$

7. Let  $X$  be a normal variable with  $\mu = 0$  and  $\sigma = 20$ . In each of the following, find  $a$ .

- (a)  $P(X \leq a) = 0.7524$
- (b)  $P(X \geq a) = 0.4268$
- (c)  $P(X \geq -a) = 0.2657$
- (d)  $P(-a \leq X \leq a) = 0.7587$

## Section 8.6

- Use normalcdf and invNorm in word problems when you know the random variable has a normal distribution.
  - When using a normal distribution to approximate a binomial probability, you need to
    - Identify  $n$  and  $p$ .
    - Compute  $\mu$  and  $\sigma$ .
    - Draw a rough sketch of the histogram to determine the area under the curve you are wanting to compute. Don't forget to add or subtract 0.5.
    - Use normalcdf to find the desired area under the normal curve.
8. The weights of newborns at a certain hospital are normally distributed with a mean of 7.2 ounces and a standard deviation of 0.75 ounces. If a newborn is randomly selected, what is the probability that their weight is more than 9.4 ounces?
9. The scores on a particular final exam were normally distributed with a mean of 75 and a standard deviation of 9. If 10% of the class made A's, 20% made B's, 35% made C's, and 25% made D's, find the cutoff point for each letter grade.
10. A fair coin is flipped 1000 times. Use the appropriate normal distribution to approximate the binomial distribution.
- (a) What is the probability of the coin landing on heads at least 520 times?
  - (b) What is the probability of the coin landing on heads between 500 and 600 times, inclusive?
  - (c) What is the probability of the coin landing on heads less than 475 times?
11. It is known that 24% of all college athletes in Texas are from out-of-state. If there are 2,000 athletes at Texas A&M, what is the probability that at least 800 but less than 1550 athletes are from Texas? Use the appropriate normal distribution to approximate the binomial probability.