

Math 166 - Spring 2008
Week-in-Review #8
courtesy: Kendra Kilmer
(covering Sections 8.4-8.6)

Section 8.4

- A **binomial experiment** has the following properties:
 - The total number of trials is fixed in advance.
 - Each trial has two outcomes: “success” and “failure”.
 - The trials are independent of each other.
 - The probability of success is the same for each trial.
- The random variable X which represents the number of successes in a binomial experiment is known as a **Binomial Random Variable**.
- If we are dealing with a binomial random variable, we must determine the number of trials (n), the probability of success in each trial (p), and the number of successes desired (r). We can then compute the probabilities by hand or on the calculator:
 - By Hand: $P(X = r) = C(n, r)p^r(1 - p)^{(n-r)}$
 - On the Calculator (Hit 2nd VARS)
 - * To calculate a single probability: binompdf(n,p,r)
 - * To add up the probabilities for r from 0 to k: binomcdf(n,p,k)
- For a binomial random variable we can easily compute statistics using the following shortcuts:
 - $E(X) = np$
 - $Var(X) = np(1 - p)$
 - $\sigma = \sqrt{np(1 - p)}$

1. Is the following random variable binomial?

- (a) An experiment consists of randomly selecting a sample of twenty oranges out of a crate containing 100 oranges of which 10 are rotten. Let Y represent the number of rotten oranges selected.

NO

probability of success changes
trials are not independent

- (b) An experiment consists of rolling a fair six-sided die five times. Let W represent the number of times the die lands on 2.

Yes

- (c) An experiment consists of randomly selecting 5 cards (with replacement) out of a standard deck of 52 cards. Let M represent the number of spades drawn.

Yes

- (d) An experiment consists of randomly selecting cards (without replacement) out of standard deck of 52 cards. Let N represent the number of cards drawn until a king is drawn.

NO

we do not have a
fixed # of trials.

2. A fair six-sided die is rolled 50 times.

(a) What is the probability of rolling a five 10 times? (Binomial RV)

$X = \# \text{ of times we roll a five}$

$n = 50$
 $p = 1/6$
 $r = 10$

$P(X=10) = \binom{50}{10} (1/6)^{10} (1-1/6)^{50-10} \approx \boxed{.1156}$
 OR// $\text{binompdf}(50, 1/6, 10) \approx \boxed{.1156}$

(b) What is the probability of rolling an even number 20 times?

$X = \# \text{ of times we roll an even number}$

$n = 50$
 $p = 1/2$
 $r = 20$

$P(X=20) = \text{binompdf}(50, 1/2, 20) \approx \boxed{.0419}$

(c) What is the probability of rolling a 3 at most 10 times?

$X = \# \text{ of times we roll a three}$

$n = 50$
 $p = 1/6$
 $r = 0, 1, 2, \dots, 10$

$P(X \leq 10) = \text{binomcdf}(50, 1/6, 10) \approx \boxed{.7986}$

(d) What is the probability of rolling an odd number at least 35 times?

$X = \# \text{ of times we roll an odd number}$

$n = 50$
 $p = 1/2$
 $r = 35, 36, \dots, 50$

$P(X \geq 35) = 1 - \text{binomcdf}(50, 1/2, 34) = \boxed{.0033}$

(e) How many times would you expect to roll an even number?

$X = \# \text{ of times we roll an even number}$

$n = 50$
 $p = 1/2$

$E(X) = np = 50(1/2) = \boxed{25}$

(f) What is the standard deviation of the number of times you roll an even number?

$\sigma = \sqrt{np(1-p)} = \sqrt{50(1/2)(1-1/2)} \approx \boxed{3.5355}$

$\text{binomcdf}(50, 1/2, 50) - \text{binomcdf}(50, 1/2, 34)$

0
1
2
3
⋮
32
34
35
36
⋮
50

3. On a given class day 15% of the students are not listening in their class. If 25 students are randomly selected,

$X = \# \text{ of students listening}$ $n = 25$
 $p = 1 - .15 = .85$

(a) What is the probability that exactly 15 of the students are listening?

$r = 15$ $P(X=15) = \text{binompdf}(25, .85, 15)$
 $\approx .0016$

(b) What is the probability that at least 16 of the students are listening?

$r = 16, 17, \dots, 25$ $P(X \geq 16) = 1 - \text{binomcdf}(25, .85, 15)$
 $\approx .9979$

(c) What is the probability that between 16 and 20 students, inclusive, are listening?

$r = 16, 17, 18, 19, 20$ $P(16 \leq X \leq 20) =$
 $\text{binomcdf}(25, .85, 20) - \text{binomcdf}(25, .85, 15)$
 $\approx .3158$

(d) How many students would you expect to be listening in the group of 25?

$E(X) = np = 25(.85) \approx 21.25 \approx 21 \text{ students}$

(e) What is the standard deviation of the number of students listening in this group of 25?

$\sigma = \sqrt{np(1-p)} = \sqrt{25(.85)(1-.85)}$
 ≈ 1.7854
 $\approx 2 \text{ students}$



4. Suppose that one-fifth of the restaurants in town are in violation of the health code. If an inspector randomly inspects seven of the restaurants,

(a) What is the probability that the first four restaurants will pass the inspection and the remaining three will fail the inspection?

the events are indep R_i be the event that the i^{th} restaurant passes

$$\begin{aligned} & P(R_1 \cap R_2 \cap R_3 \cap R_4 \cap R_5^c \cap R_6^c \cap R_7^c) \\ &= P(R_1) \cdot P(R_2) \cdot P(R_3) \cdot P(R_4) \cdot P(R_5^c) \cdot P(R_6^c) \cdot P(R_7^c) \\ &= \left(\frac{4}{5}\right) \left(\frac{4}{5}\right) \left(\frac{4}{5}\right) \left(\frac{4}{5}\right) \left(\frac{1}{5}\right) \left(\frac{1}{5}\right) \left(\frac{1}{5}\right) = \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^3 \approx \boxed{.0033} \end{aligned}$$

(b) What is the probability that only four restaurants will pass the inspection?

$X = \#$ of restaurants that pass X is binomial

$$n = 7$$

$$p = 1 - 1/5 = 4/5$$

$$r = \underline{4}$$

$$\begin{aligned} P(X=4) &= C(7,4) \left(\frac{4}{5}\right)^4 \left(1 - \frac{4}{5}\right)^{7-4} \\ &= C(7,4) \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^3 \\ &\approx \boxed{.1147} \end{aligned}$$

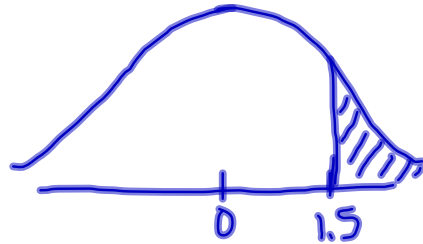
OR// use binompdf

Section 8.5

- If X is a **continuous random variable**, a **probability density function** is defined to represent the probability distribution of X . The curve lies completely above the x -axis and the total area under this curve is 1.
- A normal random variable is defined by its mean (μ) and standard deviation (σ). The probability density function associated with a normal random variable has its peak directly above the mean and is symmetric about a vertical line passing through the mean. The **standard normal variable** Z has a mean of 0 and a standard deviation of 1.
- To find probabilities associated with a Normal Random Variable, press 2nd VARS and then select option 2:normalcdf. On your homescreen, enter normalcdf(left bound, right bound, mean, standard deviation).
- If you are given a probability and asked to find a bound use 3:invNorm which is also found by pressing 2nd VARS. If you are trying to find a , then $a = \text{invNorm}(\text{total area under the curve to the left of } a, \text{ mean, standard deviation})$.

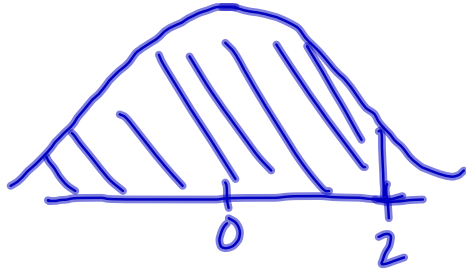
5. Let Z be the standard normal variable. Find the following:

(a) $P(Z \geq 1.5)$ $\mu=0$ $\sigma=1$

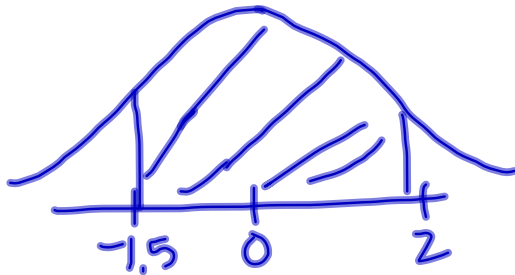


$$P(Z \geq 1.5) = \text{normalcdf}(1.5, 1E99, 0, 1) \approx \boxed{.0668}$$

(b) $P(Z < 2) = \text{normalcdf}(-1E99, 2, 0, 1) \approx \boxed{.9772}$

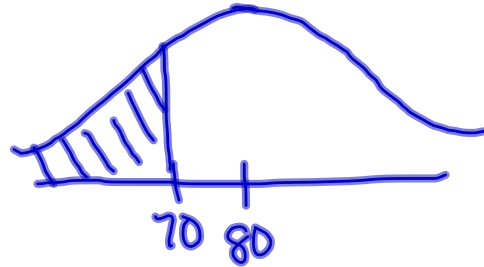


(c) $P(-1.5 < Z \leq 2) = \text{normalcdf}(-1.5, 2, 0, 1) \approx \boxed{.9104}$

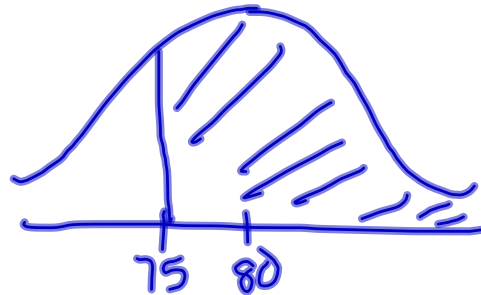


6. Let X be a normal random variable with $\mu = 80$ and $\sigma = 5$. Find the following:

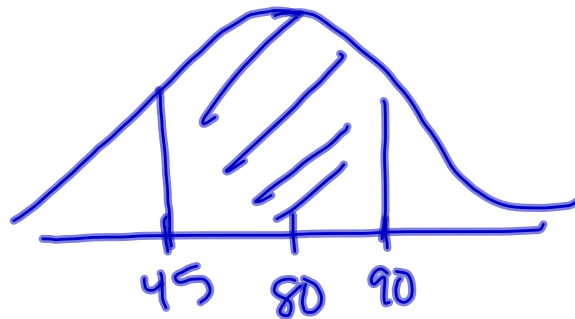
(a) $P(X \leq 70) = \text{normalcdf}(-1E99, 70, 80, 5) \approx \boxed{.0228}$



(b) $P(X > 75) = \text{normalcdf}(75, 1E99, 80, 5) = \boxed{.8413}$



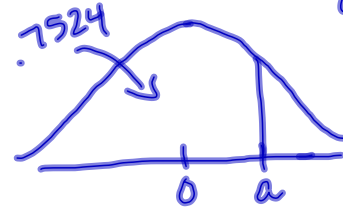
(c) $P(45 \leq X \leq 90) = \text{normalcdf}(45, 90, 80, 5) = \boxed{.9772}$



7. Let X be a normal variable with $\mu = 0$ and $\sigma = 20$. In each of the following, find

a.

(a) $P(X \leq a) = 0.7524$

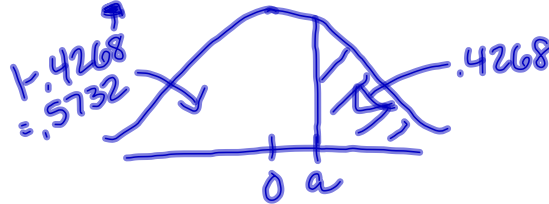


$$a = \text{invNorm}(.7524, 0, 20)$$

$$\approx \boxed{13.6412}$$

↑
total area
under curve
to left

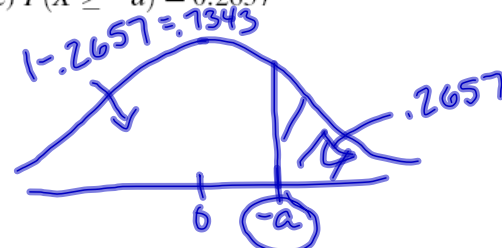
(b) $P(X \geq a) = 0.4268$



$$a = \text{invNorm}(.5732, 0, 20)$$

$$\approx \boxed{3.6905}$$

(c) $P(X \geq -a) = 0.2657$

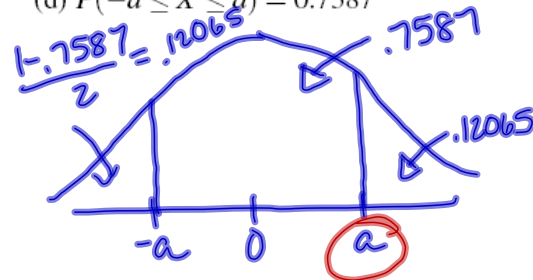


$$-a = \text{invNorm}(.7343, 0, 20)$$

$$-a = 12.5174$$

$$\boxed{a = -12.5174}$$

(d) $P(-a \leq X \leq a) = 0.7587$



$$a = \text{invNorm}(.12065 + .7587, 0, 20)$$

$$\approx \boxed{23.4349}$$

Section 8.6

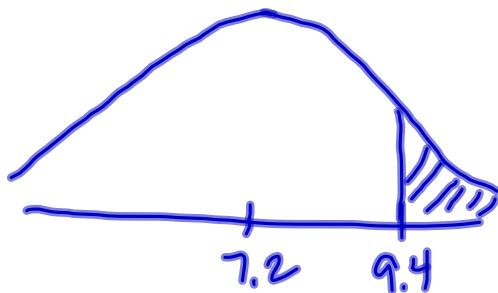
- Use normalcdf and invNorm in word problems when you know the random variable has a normal distribution.
- When using a normal distribution to approximate a binomial probability, you need to
 - Identify n and p .
 - Compute μ and σ .
 - Draw a rough sketch of the histogram to determine the area under the curve you are wanting to compute. Don't forget to add or subtract 0.5.
 - Use normalcdf to find the desired area under the normal curve.

8. The weights of newborns at a certain hospital are normally distributed with a mean of 7.2 ounces and a standard deviation of 0.75 ounces. If a newborn is randomly selected, what is the probability that their weight is more than 9.4 ounces?

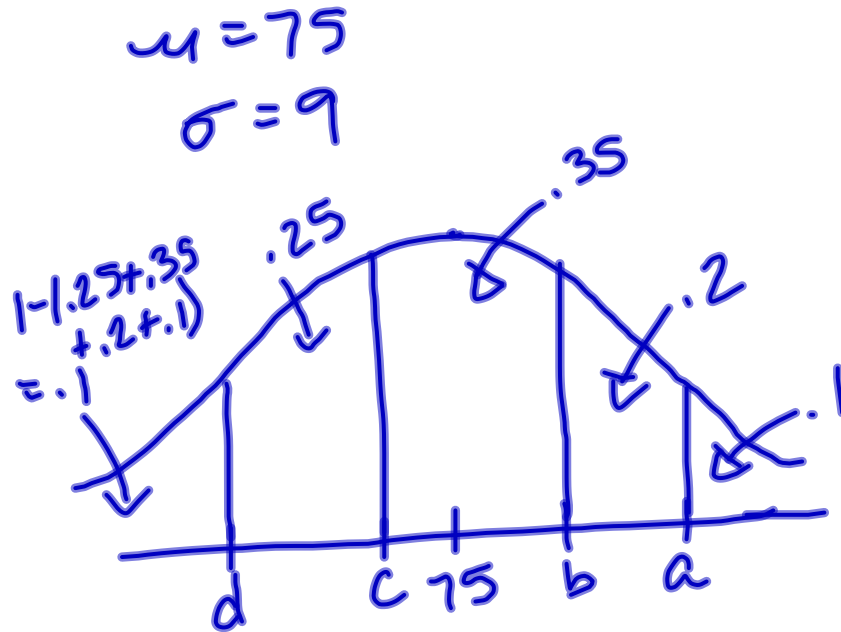
$$\mu = 7.2$$

$$\sigma = .75$$

$$P(X > 9.4) = \text{normalcdf}(9.4, 1E99, 7.2, .75) \\ \approx \boxed{.0017}$$



9. The scores on a particular final exam were normally distributed with a mean of 75 and a standard deviation of 9. If 10% of the class made A's, 20% made B's, 35% made C's, and 25% made D's, find the cutoff point for each letter grade.



$$d = \text{invNorm}(.1, 75, 9)$$

$$\approx \boxed{63} \leftarrow \text{cutoff for a D}$$

$$c = \text{invNorm}(.1 + .25, 75, 9)$$

$$\approx \boxed{72} \leftarrow \text{cutoff for a C}$$

$$b = \text{invNorm}(.1 + .25 + .35, 75, 9)$$

$$\approx \boxed{80} \leftarrow \text{cutoff for a B}$$

$$a = \text{invNorm}(1 - .1, 75, 9)$$

$$\approx \boxed{87} \leftarrow \text{cutoff for an A}$$

10. A fair coin is flipped 1000 times. Use the appropriate normal distribution to approximate the binomial distribution.

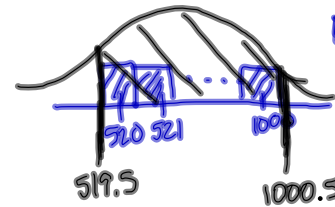
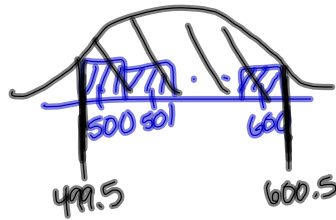
(a) What is the probability of the coin landing on heads at least 520 times?

$X = \# \text{ of times the coin lands on heads}$

$n = 1000$ $\mu = np = 1000(1/2) = 500$ $P(X \geq 520)$
 $p = 1/2$ $\sigma = \sqrt{np(1-p)} = \sqrt{1000(1/2)(1/2)} = \sqrt{250}$ $\approx \text{normalcdf}(519.5, 1000.5, 500, \sqrt{250})$
 $\approx \boxed{.1087}$

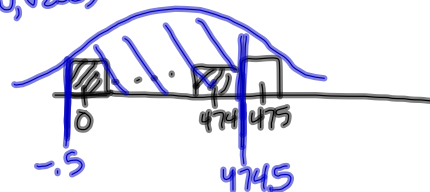
(b) What is the probability of the coin landing on heads between 500 and 600 times, inclusive?

$P(500 \leq X \leq 600)$
 $\approx \text{normalcdf}(499.5, 600.5, 500, \sqrt{250})$
 $\approx \boxed{.5126}$



(c) What is the probability of the coin landing on heads less than 475 times?

$P(X < 475)$
 $\approx \text{normalcdf}(-.5, 474.5, 500, \sqrt{250})$
 $\approx \boxed{.0534}$



11. It is known that 24% of all college athletes in Texas are from out-of-state. If there are 2,000 athletes at Texas A&M, what is the probability that at least 800 but less than 1550 athletes are from Texas? Use the appropriate normal distribution to approximate the binomial probability.

$X = \# \text{ of athletes from Texas}$

$$n = 2000$$

$$p = 1 - .24 = .76$$

$$\mu = 2000(.76) = 1520$$

$$\sigma = \sqrt{2000(.76)(1-.76)}$$

$$= \sqrt{364.8}$$

$$P(800 \leq X < 1550)$$

$$\approx \text{normalcdf}(799.5, 1549.5, 1520, \sqrt{364.8})$$

$$\approx \boxed{.9388}$$

