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To the memory of Selim Grigor'evich Krein

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To the memory of Selim Grigor'evich Krein
(1917–1998)

The 15th of July 2017 was the 100th anniversary of the birth of Selim Grigor'evich Krein, a prominent mathematician and teacher and a wonderful person.

From 1935 to 1940 he studied in the Faculty of Physics and Mathematics at Kiev University and then completed his postgraduate studies there under the supervision of N. N. Bogolyubov. The now classical result known as the Krein–Kakutani theorem dates from this period. Here 'Krein' stands for the brothers Mark (1907–1989) and Selim Krein, who proved independently of, and simultaneously with, S. Kakutani that a Banach lattice with unit 1 such that $\|x\| = \inf\{t \mid -t1 < x < t1\}$ for any vector x is isometric and structurally isomorphic to the space of continuous functions on some compact space.

Despite the large difference in age and different reactions to external hardships, the fates of Selim Krein and his older brother Mark were similar in many respects. On both of them nature bestowed remarkable mathematical talents, fantastic diligence, and a gentle and benevolent character. For a significant part of their lives both had to work at technological institutes of higher education far below their high levels of qualification instead of at a university or research institute. Yet for all their many difficulties, both created huge schools of mathematics.

During World War II Selim Krein investigated mathematical problems in the theory of cumulative charges under the direction of M. A. Lavrentiev, and in 1950 he defended his D.Sc. thesis (in the technological sciences) devoted to the motion of bodies partially filled with a fluid.

In 1954 Krein moved to Voronezh, where he worked for the rest of his life at the Technical Institute of Forestry and at Voronezh University. Here he worked very intensively as a researcher, a teacher, and a department head. There were years

when he had 10 postgraduate students and a teaching load of about 1,000 hours. The scope of his interests was very broad (fluid dynamics, operator theory, partial differential equations, numerical analysis, singular ordinary differential equations, and many other areas), but the basis of much of his research was functional analysis. Together with M. A. Krasnosel'skii he founded the Voronezh school in functional analysis, which became widely known.

Krein was one of the first researchers to apply methods of functional analysis to problems in fluid dynamics. He established fundamental results on oscillations of a viscous incompressible fluid and showed that there is a countable number of normal oscillations, which are aperiodic motions, with the possible exception of finitely many decaying oscillations. He also proved the existence of motions that decay arbitrarily rapidly and motions that decay arbitrarily slowly. The augmented two-volume version of their joint monograph with N. D. Kopachevsky and Ngo Zuy Kan was translated into English.

Problems in fluid dynamics compelled Krein to study differential equations with unbounded operators acting in Banach spaces. He investigated the question of well-posed solvability and the analyticity of solutions of the abstract Cauchy problem, indicated well-posedness classes for ill-posed problems, developed a method for fractional powers of operators, and created a theory of boundary-value problems for second-order equations. His results were presented in the book *Linear differential equations in Banach space*, translations of which were soon published in the USA and Japan. He was also the author of classical results on boundary-value problems for partial differential equations, such as an isomorphism theorem obtained in a joint paper with Yu. M. Berezanskii and Ya. A. Roitberg (simultaneously with L. Hörmander).

Krein wrote several monographs with his students (which have unfortunately not been translated into English and are difficult to find): on boundary-value problems for partial differential equations in variable domains, on boundary-value problems for overdetermined systems, and on differential equations on manifolds and Lie groups.

Along with E. Gagliardo, J.-L. Lions, and A. Calderón, Selim Krein was a founding father of the modern theory of interpolation of linear operators. He proposed a method of scales of Banach spaces which has found wide use not just in operator theory, but also in the theory of boundary-value problems for differential equations. What S.G. (as his students used to call him) valued most in the theory of interpolation of operators was its applicability to partial differential equations, fluid dynamics, the theory of waveguides, and other areas. His joint monograph with Yu. I. Petunin and E. M. Semenov on the interpolation of operators was translated into English.

It is very difficult just to list the subjects of his interest, which also included numerical mathematics, singularly perturbed differential equations, methods of commutative algebra in the theory of overdetermined systems, operator-valued analytic functions of several complex variables, and many others. For instance, in a joint paper with B. I. Korenblum and B. Ya. Levin they proposed a new (at

that time) functional analytic approach to the problem of convergence of singular integrals. In joint papers of the 1950s written with Yu. L. Daletskii for the needs of perturbation theory, they laid the foundations of the theory of double operator integrals, which was soon developed further by M. S. Birman and M. Z. Solomyak. Now this theory is again experiencing a period of extremely active development in the context of operator theory. In a joint paper with Berezanskii from about the same period, they developed a theory of hypercomplex systems with continuous basis, a theory which was forgotten for several decades, but then revived in hypergroup theory, where many of their results have been proved anew.

Selim Krein generously shared his ideas with his students and colleagues, as many examples confirm. For instance, Krasnosel'skii's classical theorem on interpolation of completely continuous operators on L^p-spaces justified a conjecture of Krein.

The handbook on functional analysis written by a team of prominent mathematicians (on the initiative of Krein, who was one of the authors and the editor) is very well known (translations were published in the USA and Poland). His very useful small book Linear equations in Banach spaces (a 'penny' book as he jokingly called it) was also translated into English.

Krein also wrote several brilliant survey papers for the journal *Uspekhi Matematicheskikh Nauk*[1] and for the VINITI series.

He was an excellent and very responsible teacher. Many generations of students remember his brilliant lectures. He wrote several textbooks: *Mathematical analysis of elementary functions* (joint with V. N. Ushakova; translated into German); *Mathematical analysis of smooth functions* (joint with N. M. Zobin), and *Mathematical programming*.

In 1967 Krein supervised the organization of the first Voronezh Winter Mathematical School, which soon became very popular and over several decades had a great influence on the development of mathematics in this country. He also took the lead in organizing both the Research Institute of Mathematics at Voronezh State University and the Voronezh Mathematical Society.

When it was known that Selim Krein was in need of heart surgery, many mathematicians in various countries were concerned. Thanks to their efforts and financial aid he was admitted to one of the leading hospitals in the USA. His return from there after the successful surgery was a festive day for Voronezh mathematicians.

Krein was given the titles of Honoured Scientist of the Russian Federation and Honoured Soros Professor. He published 170 papers and 19 books. The (probably incomplete) list in the *Mathematical Genealogy Project* contains 81 names of mathematicians who defended Ph.D. theses under his guidance and 281 names of his 'scientific descendants' at leading universities in Russia, the former Soviet Union and many other countries. Eighteen of his students earned D.Sc. degrees and two of them, Berezanskii and Daletskii, were elected full members of the National Academy of Sciences of Ukraine. A son of Ngo Zuy Kan, one of Krein's Vietnamese students and coauthors, was awarded a Fields Medal.

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[1]Translated as *Russian Mathematical Surveys*.

Krein's students will remember not only his wonderful guidance in science but also the enormous help he generously gave them in coping with 'non-scientific' matters in life: finding a job, a place to live, and so on. He was a cheerful person (regardless of circumstances) with a fine sense of humour, a bright presence in any company.

The life of Selim Grigor'evich Krein was a shining example of selfless service to people, mathematics, and fairness.

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Translated by N. KRUZHILIN