Math 308-200, Fall 2011, Assignment #2.
Due February 8th.

20 points, 1 point per a problem, except 2pt for #12.

Write your name. Provide all work. No partial credits.

Determine whether the given ODE is separable (**do not solve!**):

1. $\frac{dy}{dx} = \sin(x+y)$
2. $\frac{ds}{dt} = t\ln(s^{2t}) + 8t^2$
3. $(xy^2 + 3y^2)dy - 2xdx = 0$

Solve the following 3 equations:

4. $$\frac{dy}{dx} = \frac{1}{xy^3}$$
5. $$x\frac{dv}{dx} == \frac{1-4v^2}{3v}$$
6. $(x + xy^2)dx + e^{x^2}ydy = 0$

Solve the next three initial value problems (IVPs):

7. $$\begin{cases} \frac{dy}{dt} = (1+y^2)\tan t \\ y(0) = \sqrt{3} \end{cases}$$
8. $x^2dx + 2ydy = 0, y|_{x=0} = 2$
9. $\frac{dy}{dx} = 8x^3e^{-2y}, y|_1 = 0$
10. Sketch the solution to the initial value problem $y' = 2y - 2yt, \quad y(0) = 3$ and determine its maximum value.
11. Separation of variables in $y' = f(t)g(y)$ involves division by $g(y)$, so if for some number c one has $g(c) = 0$, one could lose $y = c$ as a solution. It is advisable to check every time whether such a possible solution is lost.

    (a) Solve the equation $\frac{dy}{dx} = (x-3)(y+1)^{2/3}$ to get

    $$y = -1 + (x^2/6 - x + C)^3 \tag{1}$$

    (b) Show that $y = -1$ satisfies the original equation.
    (c) Show that there is no value of the constant C in (1) that produces the solution $y = -1$.

12. **Radioisotopes and Cancer Detection.** A radioisotope commonly used in the detection of breast cancer is technetium-99m. This radionuclide is attached to a chemical that upon injection into a patient accumulates at cancer sites. The isotope's radiation is then detected and the site located, using gamma cameras or other tomographic devices. Technetium-99m decays radioactively in accordance with the equation $dy/dt = -ky$, with $k = 0.1155$/hr. The short half-life of technetium-99m has the advantage that its radioactivity does not endanger the patient. A disadvantage is that the isotope must be manufactured in a cyclotron. Since hospitals are not equipped with cyclotrons, doses of technetium-99m have to be ordered in advance from medical suppliers.
Suppose a dosage of 5 millicuries (mCi) of technetium-99m is to be administered to a patient. Assume the delivery time from production at the manufacturer to arrival at the hospital treatment room to be 24hr and calculate the amount of the radionuclide that the hospital must order, to be able to administer the required dosage.

In the following three problems determine whether the given equation is separable, linear, neither, or both (do not solve!):

13. $dx/dt + xt = e^x$
14. $3t = e^t \frac{dy}{dt} + yt$
15. $3r^2 = \frac{dr}{d\theta} - \theta^3$

Solve the following two equations:

16. $\frac{dv}{dx} = \frac{v}{x} + 2x + 1$
17. $\frac{dy}{dx} = x^2 e^{-4x} - 4y$

Solve the following IVPs:

18. $\frac{dy}{dx} + 4y - e^{-x} = 0, \quad y|_0 = 4/3$
19. $\sin t \frac{dy}{dt} + y \cos t = t \sin t, \quad y(\frac{\pi}{2}) = 2$