

Control and Inverse Problems for Partial Differential Equations on Graphs

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Partial differential equations on graphs (such as the wave, beam, heat, and Schrödinger equations) are used to describe many physical processes. In this talk we describe some new (and some already known) controllability results for PDEs on graphs and also some identification results. Our approach to identification (inverse) problems is based on the Boundary Control (BC) method. This method uses deep connections between controllability and identification problems and is applicable to a wide range of linear distributed systems.

Our first example is the wave equation on graphs without cycles with control applied at boundary vertices. We prove exact controllability of such systems and find a sharp estimate of controllability time which is equal to the optical diameter of the graph. Basing on these results we solve the inverse (identification) problem — recover the topology of the graph and density of the strings using boundary observations (the Dirichlet-to-Neumann map). We prove also that the exact controllability and unique solvability of the inverse problem take place if control (and observation) acts at all except one boundary vertices, however, controllability (and identification) time is bigger than in the case of the control from the whole boundary. In the case when the Dirichlet-to-Neumann map is given at a part of the boundary which does not contain two or more boundary vertices, the examples of noncontrollability and nonuniqueness of the inverse problem are constructed.

The wave equation on graphs with cycles is not exactly controllable from the boundary but may be spectrally controllable. The corresponding boundary inverse problem does not generally have a unique solution.

Finally we discuss how controllability allows to recover the spectral data using the dynamical data (Dirichlet-to-Neumann map) and how our results for the wave equation can be applied for solving control and inverse problems for the heat and Schrödinger equations.

The talk is partly based on a joint work with M. Belishev, on the paper (M.I. Belishev, *Inverse Problems*, **20** (2004), 647–672) and on Chapter VII

of the monograph (S.A. Avdonin and S.A. Ivanov, *Families of Exponentials. The Method of Moments in Controllability Problems for Distributed Parameter Systems*, Cambridge University Press, New York, 1995).