Localization on quantum graphs

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The localization theory of the quantum graphs is a natural extension of the classical theory of the random Schrödinger operators. The spectrum of the basic Laplacian of the periodic quantum graphs (similar to the necklace or the Exner's lattice) in many cases contains the point spectrum of the infinite multiplicity as well as an a.c. component. Any practical designs of such graphs necessarily accumulate some random errors (noise of the manufacturing). In 1D and quasi 1D situations the corresponding stochastic Hamiltonians demonstrate usual phenomenon of the localization (for any level of disorder). However, the standard objects of the theory (Lyapunov exponents, integrated density of states) demonstrate the new effects (in comparison with the random Schrödinger operators. In the higher dimensions (for the Exner model with random potential) the existence of localization can be proven for the appropriate high energy zones (but not everywhere). The central moment here is the existence of the arbitrary high gaps in the spectral theory of the underground periodic operators.