On the spectrum of a self-adjoint operator appearing in Smilansky's model of irreversible quantum graphs.

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In the model suggested by Smilansky one studies an operator describing the interaction between a quantum graph and a system of K one-dimensional oscillators attached at several different points $o_1, \dots o_K$ in the graph. Below x stands for a generic point in Γ (the graph) and $\mathbf{q} = (q_1, \dots, q_K)$ stands for a point in $\mathbb{R}^K$.

Mathematically, the problem reduces to the study of the operator $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$ in the space $L^2(\Gamma \otimes \mathbb{R}^K)$ generated by the diferential expression

$$\mathcal{A}U = -\Delta_x U + \frac{1}{2}\sum_{k=1}^{K} \nu_k^2\big(-\frac{\partial^2 U}{\partial {q_k}^2} + q_k^2 U\big)$$

and the conditions

$$[U_x'](o_k, \mathbf{q}) = \alpha_k q_k U(o_k, \mathbf{q}), \qquad k = 1, \dots, K$$

where $[f_x'](.)$ stands for the expression appearing in the Kirchhoff condition. The real parameter α_k expresses the strength of interaction between the quantum graph and the k-th oscillator. Note that the diferential expression $\mathcal{A}$ does not contain any parameter, but the "matching conditions", which define the domain of $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$, depend on the parameters.

One has to realize $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$ as a self-adjoint operator and to investigate its spectral properties. In spite of its seeming simplicity, the problem exhibits many unusual effects. In the talk I am going to present the results on the problem, obtained up until now.