

Properties of solutions of partial differential equations on human lung-shaped domains

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Fractal tree like structures are widely spread in nature, and for example, may be used to model the human lungs. In this work, we study the discreteness of the spectrum of Laplace and Schrödinger operators, which are defined on n -dimensional fractal rooted trees of a finite or infinite volume, and are subject to a certain mixed boundary condition. We also present a way to estimate their eigenvalues. Indeed, let $\varepsilon > 0$ denote the section diameter of the first edge of the tree. We prove that eigenvalues corresponding to such a tree are bounded from below and above by functions of the eigenvalues of appropriate operators which depend on ε and are defined on the one-dimensional skeleton of the given tree. These functions tend to the identity map as $\varepsilon \rightarrow 0$.

Since the coefficients of these one-dimensional operators depend on the section width or area of the original n -dimensional tree, we call them “width-weighted operators”. We show that the spectrum of the width-weighted operators tends to the spectrum of a one-dimensional limit operator as $\varepsilon \rightarrow 0$. The correspondence between Laplace or Schrödinger operators, and the width-weighted operators is not confined only to their eigenvalues. In particular, we prove that projections to the one-dimensional tree of eigenfunctions of the n -dimensional operators converge to those of the one-dimensional problems. Using a finite elements computer code, we investigate properties of width-weighted operators. We numerically calculate and analyze the eigenvalues and eigenfunctions of the limit operator.