

Tau Constant of Metrized Graphs

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The tau constant $\tau(\Gamma)$ is an invariant of a weighted graph introduced by M. Baker and R. Rumely. It is the value of the Arakelov-Green's function $g_\mu(x, x)$ associated to a certain "canonical measure" μ on Γ . It can be expressed in a number of other ways. In terms of spectral theory, it is the sum of the reciprocals of the eigenvalues of the Laplacian.

In this work, we prove that there is a universal lower bound, $\frac{1}{108}$, for the tau constant $\tau(\Gamma)$ of any metrized graph Γ of total length 1. We also present computational results showing that there exist graphs with $\tau(\Gamma)$ quite close to this lower bound.

Our proof of universal lower bound makes use of another representation of $\tau(\Gamma)$, which involves the resistance function on Γ . This representation is quite useful for computational purposes.

For background information on $\tau(\Gamma)$, one can consult the following references:

1. R. Rumely and M. Baker, *Harmonic analysis on metrized graphs*, submitted. Available at <http://arxiv.org/abs/math/0407427>.
2. M. Baker and X. Faber, *Metrized graphs, electrical networks, and Fourier analysis*, preprint available at <http://arxiv.org/abs/math.CO/0407428>.
3. Summer 2003 Research Experience for Undergraduates (REU) on metrized graphs at the University of Georgia. Available at <http://www.math.uga.edu/~mbaker/REU/REU.html>