NO BOOKS, NO NOTES, please. Test #1 will be based on the material of your homeworks and theory covered in class. A “MUST” material includes:

1. Derivation of weak formulation of a given strong form of boundary value problems for ordinary differential equations of second and fourth order. This will include:
   - deriving the bilinear form $a(u, v)$;
   - deriving the linear form $L(v)$;
   - characterizing the corresponding space $V$;
   - deriving the strong form from a weak form.

2. Proofs of certain facts about various norms in $V$, inequalities, etc. for $V$ being either $H^1$, $H^2$, or some subspaces of these two, e.g. $H^1_0$. This will include:
   - proving the coercivity of the bilinear form $a(u, v)$ in $V$;
   - proving the continuity of the linear form $L(v)$ in $V$;
   - Poincare inequality in $V$.

3. Proving error estimates in $H^1$ and $L^2$-norms for the FE interpolant $u_I$ and Galerkin FE solution $u_h$. This will include:
   - Cea’s Lemma and duality argument (these two are absolute must);
   - Proving error estimates of the type $\| u' - u'_I \| \leq C h^k \| u^{(k+1)} \|$ for polynomials of degree $k$, where $u_i$ is the Lagrange interpolating polynomial;
   - Proving error estimates of the type $\| u' - u'_h \| \leq C h^k \| u^{(k+1)} \|$, where $u_h$ is the Ritz-Galerkin finite element solution when using polynomials of degree $k$;
   - Proving estimates in maximum norm: $\max_x |u(x) - u_h(x)| \leq C h^k \| u^{(k+1)} \|$, where $u_h$ is the Ritz-Galerkin finite element solution when using polynomials of degree $k$;

4. FEM in multidimensional case: weak formulation of various boundary value problems for second order elliptic problems.

4. For a given boundary value problem you could be asked to compute the element stiffness or/and mass matrix, or to compute the global matrix of the Ritz-Galerkin method for a given number of finite elements.