3-D Simulation of the Pressure Drop along Horizontal Wells in a Bounded Reservoir

Richard Ewing, Akif Ibragimov, Raycho Lazarov / Institute for Scientific Computations, A&M University, College Station

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Abstract
The mechanism of pressure drop along the horizontal well and its dependence on hydrodynamic and geometrical parameters of the reservoir and the well is in the focus of our work. A model of single-phase coupled fluid flow in porous media and in a horizontal well is proposed. For solving the problem a hybrid analytical/numerical methods is tested. The developed method can be applied to analytical estimation of the influence of the various parameters (geometry of the reservoir (shape factor), length, and radius of the well, and also well/reservoir conductivity) on Production Index.

In our work we consider the whole area of drainage: the wellbore itself, bottomhole zone of the reservoir and the reservoir periphery as three adjacent media with different conductivity. For gluing different type of fluid flows with different geometry special non-overlapping domain decomposition method is developed. This method allows to reduce complicated fluid flow in 3-D media to a sequence of "simple" flows in homogenous media with standard shape (such as sphere, cylinder or annular cylinder).

Introduction
In oil recovery excessive length of horizontal wells may result in significant pressure drop along the wellbore\textsuperscript{4,16}. The mechanism of pressure drop is very complex and is due to various factors such as completion of the well, operation conditions (e.g. sand factor), the character of fluid flow inside the horizontal well and in the reservoir, geometry of the reservoir and hydrodynamic characteristic of the reservoir. These factors lead to substantial decrease of well/reservoir conductivity ratio. The pressure drop results in stabilization of the well productivity; that is, beginning with certain critical value, further exstination of wellbore's length does not incur any growth of productivity\textsuperscript{4,16}. It was already clear, and has been noted\textsuperscript{7,9,10,13,16,18}, that for the evaluation of pressure drop along the well a wellbore/reservoir flow-coupling model has to be considered.

It has been shown\textsuperscript{9} that in 3-D unbounded reservoir with permeability less than 1-D and laminar well flow the pressure drop along the wellbore is insignificant. This fact is related to the assumption that conductivity of the well in case of Poiseuille's flow is much higher than the conductivity in the reservoir and therefore, the pressure along the well changes weakly\textsuperscript{10}. At the same time, the data observed on multiple operating horizontal wells showed that productivity of these wells does not increase proportionally to the length. In recent papers\textsuperscript{13,14,16,18} this effect has been estimated by friction of the wall and acceleration.

In the present paper a model of reservoir/well system composed of a tube of small radius with extremely high (infinite) conductivity, intermediate annular zone with high but finite permeability, and the reservoir itself with low (less than 1D) permeability (Fig. 9) is studied. In the physical sense this model simulates the following situation:

- fluid flow inside tubing of the well,
- fluid flow in a screen and sand package considered as one media with its own permeability and
- fluid flow in the reservoir limited by top, bottom and external boundary.

This approach allowed to take into account main parameters that produce pressure drop along the horizontal well. It is obvious that reservoir's and well's parameters are incomparable. Therefore the joint impact of these parameters on coupled fluid flow filtration inside the well and in the reservoir could not be efficiently estimated by ordinary
methods of numerical analysis (finite element, finite differential etc.)

For this purpose two analytical approach are proposed. The first approach is based on the presentation of the reservoir pressure distribution in the form of Newtonian potential with unknown density. This discreet density is found by means of special conjugate conditions on the wall of the well.

Second approach is based on methods of separation of variables, which allowed to reduce the initial problem to the problem of computation the Fourier and Fourier Bessel coefficients on the boundaries of a simple cylindrical domain.

First approach accounts more precisely for the "local" parameters of the well/reservoir system such as significant dimensions of drainage zone, shape factors of external boundary, of the reservoir, well's length etc.

Second approach is aimed at accurate and explicit evaluation of the impact of "local" geometrical and hydrodynamic parameters such as: tubing/casing "diameter", well/reservoir conductivity ratio etc. on pressure distribution and production index.

It is very important to note that direct application of both these methods is impossible. Therefore, special domain decomposition algorithms are developed.

We also must note that for convenience these two approaches are presented in the paper separately and are applied to different domain.

**Boundary element methods for coupled flows**

In this section reservoir is modeled as a spherical layer and the well is a cylindrical cavity of constant radius and finite length. It is assumed that:

1. Inside the entire well the flow is single-phase and steady-state; the well flow is laminar, subjected Stokes-law and inertialess; the flow through the porous medium obeys Darcy's law; "constrains displacement scheme" in the wall of the porous medium and of the well is assumed to describe the cased well with very dense perforations uniformly distributed over the surface of the well bore. In this system the fluid flows in the radial direction to the interface of the well bore, and the inflow function can be considered to be continuous and smooth over the surface.

2. Then the velocity of filtration and pressure functions in the reservoir satisfies following equations:

\[
\dot{w} = -\frac{k}{\mu} \cdot \text{grad} \, P
\]

\[
LP = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left( k_i \sum_{i=1}^{3} \frac{\partial P}{\partial x_i} \right) = 0
\]

where \( x = (x_1, x_2, x_3) \) - is a point in 3-D space.

3. \( k_1 = k_2 = k_k \) and \( k_3 = k_r \) are permeability in horizontal and vertical directions correspondingly.

4. The flow region may be represented schematically by a 3-D spherical layer with a cylinder \( W \) (well) cut from it (see Fig. 1). At the top \( (x_1 = h) \) and bottom \( (x_1 = 0) \) of the reservoir the no-flow conditions are given. The pressure function is specified on the external reservoir boundary \( S (0, R) \).

5. On interface between the porous media and the well the equality condition for pressure is formulated, while for velocities in porous media and in the well allowance is made for discontinuity of the velocity component along the axis of the well.

6. In the well the pressure is given for \( x_1 = 0 \), that is, at the beginning of the horizontal well, and is equal \( P_w \). This end of the well is called dominated, and the opposite end of the well at the point \( x_1 = L \) is called the free end. The pressure at the free end is denoted \( P_L \).

7. The input data in this section are as follows: geometry of the reservoir (spherical layer with constant thickness), permeability of the reservoir in three given direction, viscosity and density of the fluid.

In it has been shown that process of coupling flow inside horizontal well and in the reservoir can be described by the following system of equations:

\[
V'(x_1) = \frac{2}{r_c} \cdot w_r(x_1),
\]

\[
- \frac{1}{\mu} \cdot \frac{dP}{dx_1} = \frac{8}{r_w^2} \cdot V'(x_1) + \frac{2}{r_w} \cdot w_r(x_1),
\]

(3)

\[
V(x_1) = P_a(x_1) + \frac{\mu}{2r_w} \left[ r_w^2 \cdot w_r'(x_1) - 4 \cdot w_r(x_1) \right]
\]

(4)

\[
P_a|_{x=0} = P_w,
\]

(5)

\[
V = \frac{1}{\mu} \int_{S(0,r_w)} k \frac{\partial p}{\partial x_1} dS, \text{ when } x_1 = L;
\]

(6)

Where

- \( S(0;r_w) \) - is a disk of the radius \( r_w \),
- \( w \) - is the velocity of filtration,
- \( P_a(x_1) \) - is the trace of pressure in the reservoir on the side surface of the well,
- \( V(x_1) \) - is the average pressure in the bore,
- \( V'(x_1) \) - is the \( l \)-st component of the average velocity,
- \( w_r(x_1) \) - is the radial component of the velocity on the lateral boundary of the well.

**Decomposition of the initial problem into complementary problems.** To solve the problem we propose the iteration method that reduces the problem of coupling fluid flow in joint system of reservoir and well into two problems- inner in the well and outer in the reservoir. This two problem are linked by means of condition (4) on the interface between well and reservoir.
Iterative scheme:
• Solve boundary dominated outer problem in the reservoir with given distribution \( P_0 \) on well surface, as a result we find the pressure function \( p(x_1) \) and \( w_r(x_1) \)-radial flow in the reservoir on the side of the well, and \( w_{x_1}(L,r) \).
• Using \( w_r(x_1) \) and \( w_{x_1}(L,r) \) solve the inner problem: (2),(3),(5), as result we obtain average pressure distribution in the well.
• Solve outer boundary dominated problem with conjugate condition (4) non-flow condition on top and bottom and given reservoir pressure on external boundary.
• Repeat until process converges.

Methods of resolving outer problem By applying the theory of Newton’s Potential, the normalized pressure function \( P(x) = \frac{P(x) - P_0}{P_0} \) can be represented in the following discrete form (see Fig. 2):

\[
P = \int G(x, \xi) \beta(\xi) d\xi \approx \sum \beta_i \int G(x, \xi) dL_i
\]

where the unknown discrete densities distributions are found from the equation

\[
P(x_j) = P_w \quad \text{when} \quad x_j \in \partial \Omega
\]

and \( G(x; \xi) \) is the Green’s function of the mixed problem with non flow conditions on top and zero pressure on the internal boundary. In the simplest isotropic case, when \( k_1 = k_2 = k_3 \) and \( k_2 = k_3 \) are the permeability in horizontal and vertical directions correspondingly the Green’s function is singular positive solution of the problem:

\[
\Delta G(x, \xi) = 0 \quad \text{in} \quad \Omega \setminus \xi;
\]

\[
\frac{\partial G}{\partial x_1} = 0 \quad \text{when} \quad x_1 = 0 \quad \text{and} \quad x_1 = l;
\]

\[
G = 0 \quad \text{on} \quad S(0, R_0)
\]

In an infinite reservoir of finite thickness \( R_0 = \infty, h = 1/2 \) the function \( G(x; \xi) \) is a superposition of infinite series of Green functions' These series are diverging in a general sense, and they are transformed in \( 1 \) by means of specific asymptotic decompositions. This method is not applicable in case of a bounded reservoir \( R_0 < \infty \), consequently, an iterative method for construction of this type of the Green functions was developed in \( 17, 22 \). The main idea consists to build of a special control condition on the sphere, such that “source” flow generated under that conditions in the well will satisfies non-flow conditions on top and bottom.

The results and discussion. As noted above, have been interested in the dependence of pressure drop and local production distribution along the well on geometrical parameters of reservoir-well system. It has been shown \( 8 \), that under the assumption of the flow obeying conditions “1-8.” in unlimited 3-D porous media with “normal permeability”(less than 1D), pressure drop along the wellbore is insignificant. In bounded reservoir dependence on geometrical parameters could significantly affected on pressure drop (see, e.g.\( 8 \)).

Examine the impact of geometrical characteristics of reservoir boundary on production rate and pressure drop is the aim of this section.

Here the variable parameters are: \( L \) - length of the well, \( D_b \) - distance between free end of the well and reservoir boundary; location of the dominated end of the well, \( r_w \)-well’s radius, shape of the reservoir boundary namely \( 1/R_b \)-radius.

Our computations show that pressure drop along the wellbore depends essentially on the shape of the reservoir boundary. The results for two limiting cases (spherical and plane external boundaries) are given in Fig. 4. Here \( P < P_w \) is the average pressure in the wellbore and \( P_w \) is the pressure at the fixed (boundary dominated with given pressure) end. In both cases the distance to the external boundary is the same. This results shows, that boundary shape could have up to 10% increase of pressure drop.

Next we show the impact of the distance between well and reservoir boundary on the pressure drop in the well. It is clear that if the free end of the well intersects with the reservoir external boundary, the pressure drop is identical to the differential pressure. We studied the influence of the distance to the external reservoir boundary on value of the pressure at the free end of the well. Computational results, under assumption of constant well length, reservoir’s and well’s radiiuses are presented in the Fig. 3. Here \( P_L \) is the pressure at the free end of the wellbore. As may be seen from the figure this relation is not significant till distance \( D_b \) is enough big (in this run about \( 20r_w \)). But when the free end of the well is enough close to external boundary the pressure at this end decrease exponentially.

It is worth noting that the increase of length, which does not decreases the distance \( D_b \), also causes considerable pressure drop at the free end of the well.

For this purpose the model of reservoir with fixed distance \( D_b \) and enough large \( R_b \) has been considered. Variable parameters are the length of the well. Corresponding results of simulator runs are presented in the Fig. 5. These results show that the influence of the dominated end of the well on the free end of the well could decrease even proportionally to the well length.

It is known that as a result of pressure drop along the well the dependence of production rate on the well length tends to be constant. So, the relation between the production rate of HW and distance to the reservoir external boundary has been studied for two cases: \( 1 \)- constant pressure in the well and \( 2 \)-variable pressure in the well generated by reservoir/well flux. As shown in Fig.6, in case of constant pressure the production rate sharply decreases and stabilized as the distance from reservoir boundary increase. In case of variable pressure in the hole, the curve is much smoother.

As it is noted above the shape of the reservoir could result in substantial changes of pressure drop. In considered model a quantitative characteristics of the shape of the reservoir boundary is the curvature radius \( R_a = 1/R_b \). The production rate also significantly depends on \( R_e \). The results of the
computational run with fixed length, radius of the well, and
distance $D_0$ and variable $R_e$ is illustrated in Fig. 7. According
to this result there exist such a critical value $R_{w0}$, depending on
$L$ and $D_0$, that if $R_e$ is larger than this critical value $R_{w0}$ then
the production rate would no longer depend on $R_e$. It is important
to note that all these results were obtained for abnormal high
permeability values ($k=10^{D}$), very short distances ($10m$) to
the external reservoir boundary and enough big radius of the well
($0.1m$). We wanted to show that the proposed method could be
used to estimate elaborate qualitative effects of the system
geometry on the production rate and the pressure distribution
along wellbore. When the distance to the external boundary and
the reservoir permeability were chosen more realistically,
the qualitative picture is the same but quantitatively the
pressure drop is significantly smaller. Thus, we came to the
same conclusion as in, that within the considered
assumption there are no limitation on the length of horizontal
well imposed by its hydrodynamics effectiveness.

At the same time it is well known that the hydrodynamics
resistance of a pipe strongly depends on its radius. For this
purpose the impact of the well bore radius on the pressure
drop along the well has been studied. The results of run under
the assumption “1-8” are presented in the Fig. 8. In these runs
we varied the well radius, and fixed the parameters
$R_0=10000m$, $k=100MD$, $D_b=100m$, $L=1000m$. As shown in
Fig. 8, the pressure drop in a well of very small radius may
reach tens percents. High-pressure drops in a long horizontal
well may thus be accounted for by different technological
reasons (such as well’s completion, casing/tubing diameters,
sand factor, screen permeability etc.) resulting in decrease of
the actual diameter of borehole. Model and method that can
take into account the impact of these parameters on the
pressure drop are presented in next section.

Separation of variables for coupling flow model in
cylindrical media

In this section the well is modeled as a super-collector in
cylindrical porous media with extremely high permeability.
Corresponding mathematical problems are solved by the
method of separation variables and the solutions are
represented in the form of cylindrical functions. We consider
the following “cylindrical layer” (Fig. 9) model:

Well=$A_0+A_1$,

$A_0=\{y^2+x^2<r_0^2; 0<z<L\}$;

$A_1=\{r_0^2<y^2+x^2<r_w^2; 0<z<L\}$

Reservoir=$A_2+A_3$

$A_2=\{r_w^2<y^2+x^2<R_0^2; h<y<H; 0<z<L\}$

$A_3=\{0; h<y<H; L<z<B_0\}$

This complex cylindrical region seems suitable for the
representation of conjugate fluid flow in HW-Reservoir

system. One of the possible treatment of coupling filtration in
this complex region $A=A_0+A_1+A_2+A_3$ is the following:

$A_0$—tubing or more general — oil or gas transport domain,
where flow subjected pipe hydrodynamics;

$A_1$—screen or more general — super collector: a high-
permeability bottom hole zone with small diameter;

$A_2+A_3$—reservoir by itself, limited by top ($y=H$), bottom ($y=h$).

This complex domain contains also boundary of
discontinuity of the media, where corresponding conjugate
conditions are satisfied. Thus we consider the model:

In $A_0$ — average pressure $P_a$ satisfies equations (1-3), and
boundary conjugate conditions (4) on the side surface
$y^2+x^2=r_0^2$;

In $A_1$, $A_2$ and $A_3$ the pressure $P(x,y,z)$ satisfies Laplace
equations

The pressure function and normal component of the
filtration velocity on the interfaces between these three regions
satisfies continuity conditions.

Analytical solution of this general problem is very
difficult, but possible due to the developed iterative
techniques. First, it has been shown that there exists such
pressure distribution on the external boundary, that the
solution of the problem in a cylinder with this boundary
condition will satisfy non-flow conditions on top and bottom.
Therefore, the problem at the first step is reduced to a problem
in a domain containing three embedded into each other
cylinders. Then the problem is reduced to two problems in
heterogeneous cylinders $A_0+A_1+A_2$ and homogeneous
cylinder $A_3$. This reduction makes possible on the next step to
apply the non-overlapping domain decomposition algorithm.

This allows to represent the pressure in imbedded into each
other cylinders as a limit of sequence of solutions defined in
$A_0+A_1+A_2$ and in $A_3$, correspondingly. This sequence is
linked through special conditions on the interior boundary $z=L$.

The approach presented in the first section allows to
reduce the problem of filtration in $A_0+A_1+A_2$ to a sequence
of problem in $A_0$ and $A_1+A_2$. This sequence is linked
through conjugate condition (3) and converges very fast.

Finally, the overall problem is reduced to the problem of
filtration with mixed boundary conditions in heterogeneous
porous media in an annular cylinder $A_1+A_2$ with
permeability $k_1$ in $A_1$ and $k_2$ in $A_2$. Detailed description of
the proposed methods is beyond the scope of the present
paper. Here we will cover part of the algorithm that gives the
analytical solution of the last problem in highly heterogeneous
porous media.

The algorithm is realized in the form of Fourier Bessel
series. It makes possible to investigate explicitly the
dependence of the pressure drop on the well radius
(casing/tubing diameter) and well/reservoir conductivity ratio.

Domain decomposition in highly heterogeneous
annular. In this section reservoir/well system is modeled as
annular cylinder $A_1+A_2$. We introduce new variables $x=\frac{z}{r_w}$,
$y=y/r_w$, $z=z/r_w$ then the main dimensionless geometrical parameters are: $L=L/r_w$, $R_0=r_0/r_w$, $R=R_0/r_w$. Domains $A_1$ and $A_2$ are transformed into:

$$A_1 = \{r_0 < r < l; 0 < z < L\} \text{ and } A_2 = \{l < r < R; 0 < z < L\}$$

It is assumed that:
- We have Darcy flow with permeability equal $k_i$ in $A_1$ and $k_j$ in $A_2$.
- On the interior interface ($r=I$) between $A_1$ and $A_2$, the pressure function $P$ and influx (the radial component of velocity) are continuous.
- The pressure function is given on boundary dominated end $z=L$, $r < I$ and equal $P_w$.
- Non flow condition is specified (in sense of symmetry) for $z=L$ and $I < r < R$.
- Reservoir pressure $P_b$ on the left side of the cylinder ($z=L$) and on the external boundary ($r=R$) is given.
- Pressure for $r=R_0$ is a linear function of variable $z$ (more general distribution could be considered in the same way as in previous section).

Thus for reduced pressure function

$$P = (P - P_b)/(P_w - P_b)$$

we obtain the following boundary problem:

$$\text{div}(\text{grad} P) = 0 \quad \text{in} \quad A_1 \quad (11)$$

$$P_1 = I \quad \text{when} \quad z = L \text{ and } R_0 < r < R \quad (12)$$

$$P_1 = z/L \quad \text{when} \quad r = R_0 \quad (13)$$

$$\text{div}(\text{grad} P_2) = 0 \quad \text{in} \quad A_2 \quad (14)$$

$$\frac{\partial P_2}{\partial z} = 0 \quad \text{when} \quad z = 0 \text{ and } I < r < R \quad (15)$$

$$P_2 = 0 \quad \text{when} \quad r = R \text{ and } 0 < z < L \quad (16)$$

$$P_2 = 0 \quad \text{when} \quad z = L \text{ and } I < r < R \quad (17)$$

Conjugation conditions on the interface between the media $A_1$ and $A_2$ are following:

$$P_1|_{r=I} = P_1|_{r=I} \quad (18)$$

$$k_1 \frac{\partial P_1}{\partial r} |_{r=I} = k_2 \frac{\partial P_2}{\partial r} |_{r=I} \quad (19)$$

Solution method. It is clear that if coefficient of heterogeneity $G = k_2/k_1$ is extremely small then pressure in reservoir is close to discontinuous function that is a linear function in $A_1$ and equal 0 in $A_2$. And when $G$ increases then this pressure function might be adjusted both in $A_1$ and in $A_2$ and become smoother. Moreover it is obvious that the increase of $G$ decreases the pressure in $A_1$ and increases the pressure $A_2$. This process is implemented as following iterative procedure:

- Solve at each step two problems: in the domain $A_1$ with homogeneous Dirichlet condition on the side $r=I$ and in the domain $A_2$ with Neuman conditions on the same side.
- Extend through the interface between $A_1$ and $A_2$. The extension is subjected conjugate condition (18)-(19).
- Correct this extension on non-joint parts of a boundary. A distinctive feature of this process is the correction of the solution only on non-joint part of boundary, because on the interface between $A_1$ and $A_2$ the pressure and normal flux satisfy conjugate conditions on each step automatically.

From a mathematical point of view this iterative procedure make possible compose the solution of the problem (11-19), as a sum of three type of functions: $U_i(r,z)$, $W_i(r,z)$ and $V_i(r,z)$, where $U_0(r,z) = zL$, and for $i = 1, 2, ...$, function $U_i(r,z)$ is a solution of problem:

$$\text{div}(\text{grad} U_i(r,z)) = 0 \text{ in } A_1 + A_2$$

$$U_i(R_0, z) = 0 \quad (20)$$

$$U_i(r, 0) = 0 \quad (21)$$

$$\frac{\partial U_i(I, z)}{\partial r} = 0 \quad (22)$$

and satisfies following penalty conditions on $z = L$

$$\frac{\partial U_i(r, L)}{\partial z} = -\frac{\partial V_i(r, L)}{\partial z}, \text{ when } R_0 < r < I \quad (23)$$

In addition, function $V_i(r, z)$ for $i = 1, 2, ...$ is solution of the problem:

$$\text{div}(\text{grad} V_i(r,z)) = 0 \text{ in } A_1 + A_2$$

$$V_i(1, z) = V_i(R, z) = 0 \quad (24)$$

$$V_i(0, z) = 0, \quad (25)$$

and satisfies penalty conditions on $z = L$, when $I < r < R$

$$\frac{\partial V_i(r, L)}{\partial z} = -\frac{\partial U_{i-1}(r, L)}{\partial z} \quad (26)$$
Moreover, function $V_i(r,z)$ must satisfy conjugate conditions (18-19). In the Appendix we show that such function exists and satisfies the conditions:

$$V_i(0,0,z) = 0$$

and

$$\frac{\partial V_i}{\partial r}(1-0,0) = G \frac{\partial V_i}{\partial r}(1+0,0)$$

(29)

It is not difficult to see that function

$$U_0 + \sum U_i + \sum V_i$$

satisfies all conditions except on $r=1$

For this purpose following correction function is introduced:

$$\text{div}(\text{grad} W_i(r,z)) = 0$$

$$W_i(r,z) = 0$$

(31)

$$W_i(r,0) = 0$$

(32)

$$W_i(1-0,z) = W_i(1+0,z)$$

(33)

$$\frac{\partial W_i}{\partial r}(1-0,z) = G \frac{\partial W_i}{\partial r}(1+0,z)$$

(34)

$$\frac{\partial W_i}{\partial z}(r,0) = 0, \text{ when } R_0 < r < R$$

(35)

And penalty condition on the cylinder side $r=R_0$

$$W_i(R_0,z) = -V_i(R_0,z)$$

(36)

It is shown (see Appendix) that these functions exist and can be represented in the form of Bessel function, and modified Bessel function of zero order and first and second kind. Moreover, it is shown that there exists such condition on parameters $G$, and $R/R_0$, but not on the length $L(1)$, that $V_i(r,z)$ tends to zero with rate $q^i (q < 1)$.

Then the function

$$u_N = U_0 + \sum_{i=1}^{N-1} (W_i + U_i) + \sum_{i=1}^{N} V_i$$

(37)

satisfies all conditions except conditions (12) on the side $r=R_0$ and (13) on the end $z=L$ and $R_0 < r < 1$ where $V_i(r,z)$ tends to zero. Thus $u_N$ tends to the solution of the solution of the problem (11-19).

Presentation (37) contains main pressure function $U_0$ generating flow in the pipe (solution of the problem (2-5) and sum of terms reflected for perturbations of reservoir's flux. These terms dominate in case when $G$ is near 1 and are negligible when coefficient of heterogeneity $G$ tends to 0. Corresponding results are presented on Fig.11.

Conclusions:

1. The actual ratio reservoir/well conductivity has a greater impact on the pressure drop as compared to other parameters of the system "reservoir-HW". Second in significance parameter influencing the pressure drop is the radius of the well.

2. The ratio reservoir/well conductivity is defined by the completion of the well (tubing radius, actual "screen+and pack" conductivity etc.) and actual radius of oil/gas flow along the tubing.

3. When the ratio of reservoir/well conductivity is very small ($G<<1$), then the productivity index of horizontal well is significantly affected by the geometrical parameters of the reservoir. From our results it can be deduced in the case $G<<1$ that: the distance from the external boundary on the first place, the shape factor (curvature of the reservoir boundary) on the second place, and length of the well in third place affect the pressure drop along the well.

4. Because of the pressure drop along the wellbore, the productivity of HW, beginning at certain critical value ceases to grow with increase of its length. The proposed computational methods can be used to predict accurately this critical length and its dependence on ratio reservoir/well conductivity.

Nomenclature

$\mu$ = viscosity
$R_w$ = radius of the well
$L$ = length of the well
$<P>$ = average pressure in the wellbore
$P_i$ = is the pressure in the fixed (dominated) end of the well
$P_f$ = is the pressure on the free boundary of the well
$D_0$ = distance between free boundary of the well and reservoir boundary
$R_e$ = radius of external boundary
$h$ = reservoir thickness
$R_e = 1/R_b$ = curvature radius of external boundary (shape factor)
$S(0,R_b)$ = sphere of radius $R_b$
$r_q$ = casing radius
$r' = \sqrt{x^2 + y^2}$
$J_0(r_0)$ = Bessel function of zero order of first (second) kind.
$I_0(K_0)$ = Modified Bessel functions of zero order of first (second) kind.
\[ \text{div(\text{grad } u)} = \frac{\partial^2 u}{\partial z^2} + r \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \]

**Reference**


**Appendix-Presentation of the Functions $W, U$ and $V$**

The first approximation $U_0$ is a linear function $z/L$ and it satisfies whole conditions except two boundary conditions: (15) when $z=L$, $1<r<R$, and (16) when $r=R, 0<z<L$. To correct initial function $U_0$ problem (24 – 26) is solved. The solution of the problem (24 – 26) is sought in form of Fourier Bessel Series 22.24:

\[ V = \Phi_i(r, z) \text{ in } A_1, \text{ and } V = G \Phi_i(r, z) \text{ in } A_2. \]

Here

\[ \Phi_i(r, z) = \left\{ \sum_{i}^{\infty} R_m(r) D_m(i) \frac{\sinh(v_m z)}{v_m L} \right\} \]

\[ R_m(r) = J_0(v_m r) Y_0(v_m r) - J_0(v_m r) Y_0(v_m r) \]

The $v_m$ in (A-1) and in (A-2) is the root of the equation $R_m(R)=0$. The $D_m(i)$ in (A-1) is a Fourier Bessel coefficient of the function $-1/L$.

The function $U_0+V_1$ satisfies all conditions except condition (12), (13) and (16). To correct this function problems (20-23) and (31-36) with corresponding boundary conditions (23) and (36) are solved. Solution of the problem (20-23) can be represented in the form:

\[ U_i(r, z) = \sum_{k=1}^{\infty} C_k(i) E_k(r) \frac{shv_k z}{shv_k L} \]
In (A-3):
\[ E_k(r) = J_0(\mu_m r)Y_0(\mu_m R_0) - J_0(\mu_m R_0)Y_0(\mu_m r) ; \]
\[ \mu_m \]

is a root of equation \[ E_k(R)=0 ; \]

\[ C_k(i) \]

are Fourier Bessel coefficients of the function \[ -G \Phi_i(r, L) \]. It is not difficult to see that \( C_k^* i \)

exist, such that \[ C_k(i) \leq C_k^* i \]

(A-4)

and, satisfies the equation \( C_k(i+1) = -G C_k^* i \sum_{m=1}^{\infty} g_m Q_m \).

(A-5)

Here

\[ Q_m = \frac{2}{1 - \frac{R_0}{R}} \int_{R_0}^{R} (R_m(r)r) dr \]

and

\[ g_m = \frac{\pi J_0(v_m R)}{J_0(v_m) + J_0(v_m R)} \]

Suppose that \( G, R \) and \( R_0 \) are such that \[ G \sum_{l=1}^{\infty} Q_m g_m \leq q \leq 1 \]

(A-5).

From (A-4)-(A-6) follows that \[ C_k(i) \leq q^l \]

And therefore tends to 0 with the rate of geometric progression.

Our next and last correction concerns conditions on the side of cylinder \( RJR \), and \( r=R_0 \).

For this purpose the problem (31-36) is introduced. The solution of this problem is composed from modified Bessel function of first and second kind and has a form:

\[ W_1(r, z) = \sum_{n=1}^{\infty} R_n^1(r, z) \sin(B_n z) \text{ In A1} \]

\[ W_1(r, z) = \sum_{n=1}^{\infty} R_n^2(r, z) \sin(B_n z) \text{ In A2} \]

Here:

\[ R_n^1 = a_n^1 I_0(B_n r) + a_n^2 K_0(B_n r) \]

\[ R_n^2 = b_n^1 I_0(B_n r) + b_n^2 K_0(B_n r) \]

\[ B_n = \frac{2n-1}{L} \pi \]

The coefficients \( a_n^1, a_n^2, b_n^1, b_n^2 \) are obtained from the algebraic equations:

\[ a_n^1 I_0(B_n) + a_n^2 K_0(B_n) = b_n^1 I_0(B_n) + b_n^2 K_0(B_n) \]

\[ a_n^1 I_1(B_n) - a_n^2 K_1(B_n) = G(b_n^1 I_1(B_n) - b_n^2 K_1(B_n)) \]

\[ a_n^1 K_0(B_n R_0) = f_n^1 - a_n^1 I_0(B_n R_0) \]

\[ b_n^1 I_0(B_n R) = f_n^2 - b_n^2 K_0(B_n R) \]

Here:

\[ f_n^1 \text{ - Fourier coefficient of } V_i(R_0, z) \]

\[ f_n^2 \text{ - Fourier coefficient of } U_{i,1}(R, z) \]

Then the function

\[ u_i = \sum_{l=0}^{N} U_i(r, z) + \sum_{i=1}^{N} V_i + \sum_{i=1}^{N} W_i \]

satisfies whole conditions except conditions on annular \( z=L_1 \), \( r<R \) where \( U_i(r, L) \) tends to 0 because of condition (A-5). Q.E.D.

![Fig. 1. Scheme of the reservoir](image)
Fig. 2-The well model.

Fig. 3-Dependence at the free end on the distance to reservoir external boundary.

Fig. 4-Distribution of the relative average pressure within a well in reservoir with plane (1) and spherical (2) external boundary.

Fig. 5-Dependence of the pressure on HW length.

Fig. 6-Relation between the production rate of HW and the distance to the reservoir external boundary.
Fig. 7 - Relation between the production rate of HW and curvature radius of reservoir external boundary.

Fig. 9 - Scheme of cylindrical layer in the plane $(x, y, \theta)$.

Fig. 10 - Scheme of cylindrical layer in the plane $(x, \theta, z)$ in radial coordinate $(r, \theta)$, $0 < r < R_b$, $0 < z < L$.

Fig. 8 - Pressure drop along the wellbore vs. borehole radius.