MATH 152H, FALL 2005, PRACTICE TEST I

Disclaimer: This practice test DOES NOT serve as an indication of the contents of the actual test. It only suggests a possible format.

Please print your name and your section number clearly!

Name:_____ Section:_____

Please show all your work, that is explain every step of your solution - it is your work, not the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Cheating is strictly prohibited by the University's Code of Honor, and will be prosecuted. Good luck!

Problem 1. Let $f(x) = \frac{cos(x)}{sin(2x)}$ and $g(x) = -\frac{sin(x)}{sin(2x)}$.

a) Find the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$ on the interval from $\frac{\pi}{4}$ to $\frac{\pi}{3}$.

b) Find the volume of the solid obtained by rotating the region bounded by $h(x) = \frac{f(x)}{g(x)}$, x-axis, and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$ about the x-axis.

Problem 2. Let $f(t) = e^t\sqrt{9 - e^{2t}}$, and for each x on the interval $[0, 1]$ define $g(x) = \int_0^x f(t)dt$.

a) Find an explicit expression for $g(x)$. Find $g'(x)$ using Fundamental Theorem of Calculus.

b) Prove that there exists a number c in the interval $[0, 1]$ such that $f(c) = g(1)$.

c) Would the statement of part b still hold on the interval $[0, 2]$? Explain.

Problem 3. Let S be the region in the xy-plane enclosed by the curves $f(x) = \frac{1}{x(x+1)}$, $g(x) = \frac{x}{2}$, and the line $x = 2$. Let V be a solid whose base is S so that cross-sections perpendicular to the x-axis are triangles with base lying in S and height 1.

a) Sketch S. Explain.

b) Find the volume of V.

Problem 4. Take a cube of sidelength R and drill two cylindrical holes of radius $r < R$, each perpendicular to two opposite faces, so that lines through the centers of the two cylinders intersect at the right angle.

a) Find the volume of the resulting solid.

b) Would the volume in part a change if the lines intersected at a different angle, or did not intersect at all? Explain.