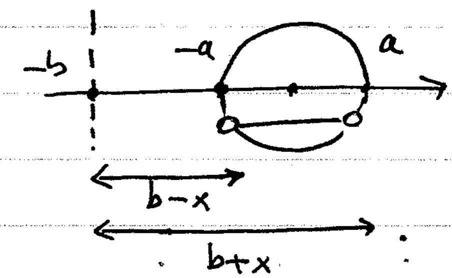
Problem B & C. For $0<a<b$ rotate $x^2+y^2=a^2$ about $x=-b$

1^{st} The Volume (by Washer Method)

$$V=\int_{-a}^{a} \pi(b+x)^2-\pi(b-x)^2 \, dy$$



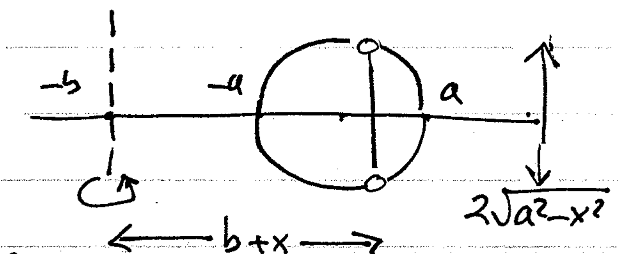
$$=\int_{-a}^{a} \pi(2b)(2x)\,dy$$

$$=(2\pi b)\int_{-a}^{a} 2\sqrt{a^2-y^2}\,dy = (2\pi b)(\text{Area of circle of radius } a)$$

$$=(2\pi b)(\pi a^2)$$

2^{d} The Volume (by Shells Method)

$$V=\int_{-a}^{a} 2\pi(b+x)\left[2\sqrt{a^2-x^2}\right]dx$$



$$=2\pi b\int_{-a}^{a} 2\sqrt{a^2-x^2}\,dx + 4\pi\int_{-a}^{a} x\sqrt{a^2-x^2}\,dx$$

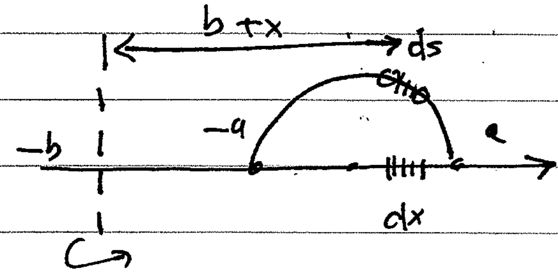
$$=2\pi b\begin{pmatrix}\text{Area of circle}\\ \text{of radius } a\end{pmatrix} + (4\pi)\left(\tfrac{2}{3}\right)(a^2-x^2)^{3/2}\left(\tfrac{-1}{2}\right)\Big|_{x=-a}^{x=a}$$

$$=(2\pi b)(\pi a^2)+0$$

3d Surface Area

For $y=\sqrt{a^2-x^2}$, $ds=\dfrac{a}{\sqrt{a^2-x^2}}dx=\dfrac{a}{y}dx$

is the differential of surface area on the top half of $x^2+y^2=a^2$



$\frac{1}{2}(SA)=$

$\int_{-a}^{a} 2\pi(b+x)\frac{a}{y}\,dx=$

$(2\pi b)\int_{-a}^{a}\frac{a}{y}\,dx+(2\pi a)\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}\,dx$

$=(2\pi b)\left(\text{half the perimeter of circle of radius } a\right)+(2\pi a)\,2\,(a^2-x^2)^{1/2}\left(\frac{-1}{2}\right)\Big|_{x=-a}^{x=a}$

$=(2\pi b)\left(\frac{1}{2}\right)(2\pi a)+0$, so

$SA=(2\pi b)(2\pi a)$