

Math 128a, Homework 2

due September 18. Also due: problem 4 from last time.

1. Let $x_i = i$ for $i = 0, 1, 2, 3, 4$. Let f be the function such that $f(x) = x^3$ for $0 \leq x \leq 2$, $f(x) = 12x - 16$ for $x \geq 2$. Let $f_i = f(x_i)$, i.e. $f_0 = 0, f_1 = 1, f_2 = 8, f_3 = 20, f_4 = 32$. Let $\bar{x} = 1.5$.

- Calculate an approximation to $f(1.5)$ using Neville's algorithm.
- Calculate the interpolating polynomial using divided differences, and use it to approximate $f(1.5)$.
- In addition, assume that it is known that $f'(1) = 3$. Calculate the corresponding Hermite interpolating polynomial, and use it to approximate $f(1.5)$.
- Interpolate f by a function that is linear on each interval $[x_i, x_{i+1}]$, $i = 0, 1, 2, 3$. Use it to approximate $f(1.5)$.

For each of the above methods, calculate the true absolute error. Calculate the theoretical bound on the absolute error when such a bound makes sense. On a single plot, graph the true curve and the interpolating curves obtained in the last three methods. Comment on the relative quality of the interpolations.

2. Let f be an n times differentiable function. Let $x_0 \leq x_1 \leq \dots \leq x_n$; note that some of the x_i 's may be equal. Prove that for some $\xi \in [x_0, x_n]$,

$$f[x_0, x_1, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}.$$

3. Download the file `pchip.m` from the class webpage; a copy of this file is also available in the directory "public" in the root directory on socrates. The file contains a Matlab 6 function for Hermite interpolation. Note that this function is different from what we were considering in class, since it does not take as an argument the values of the derivatives of the original function. Describe what this function does, in particular, exactly how the derivatives of the interpolating polynomials are chosen.

Modify the function to take an extra argument, namely the values of the derivatives of the original function at the knots. Rename the function `pchip2.m`, and modify it to produce an interpolating function which is a cubic polynomial in each interval between the successive knots, and which takes the prescribed values and derivatives at the knots. Test this function, and `pchip.m`, on the data in Problem 4 from the preceding homework. In the last part, take $(1, 1/2, 1/3, \dots, 1/6, 4/7, 1, 1, \dots, 1)$ as the values of the derivatives at the knots. For each case, plot the true function (on the given interval), and the two interpolating functions. Discuss the level of precision.

4. Exercise 2.12 of the textbook.