Math 128a, Homework 6
due October 23.

1. Problem 4.1 (a-c).

2. (a) Problem 4.3(a).
(b) Suppose that we have done Gaussian elimination on a matrix $A$, so that solving $Ax = b$ for a new $b$ costs just $\text{const} \cdot n^2$ operation, since we can reuse $A$’s $L$ and $R$ factors. Let $u, v$ be vectors, and denote by $v^T$ the transpose of $v$, and by $\cdot$ the dot product. Show that the following algorithm solves $(A + u \cdot v^T)x = b$, and takes $\text{const} \cdot n^2$ operations. This is much cheaper than Gaussian elimination on $A + u \cdot v^T$, which would cost $2/3n^3$. This is useful because it is common to have to solve several different systems of linear equations where the matrices differ by just adding $u \cdot v^T$ for some column vectors $u, v$. Hint: use part (a).

1) Solve $Az = b$ for $z$.
2) Solve $Ay = u$ for $y$.
3) Compute $\alpha = v^T y$ (a dot product).
4) Compute $\beta = v^T z$ (a dot product).
5) Compute $x = z - \frac{\beta}{1+\alpha} y$ (adding a multiple of one vector to another).


4. Write a function to decompose a matrix in its LU decomposition. Define random matrices with dimensions $10 \times 10$, $100 \times 100$ and $1000 \times 1000$, and compare the speed of your function with that of the built-in function `lu.m`. For the $10 \times 10$ matrices, compare the matrices themselves. Describe your observations.