

Math 128a, Homework 7

due October 30.

1. Problem 5.2.

2. Problem 5.3.

3. Problem 5.5.

4. Suppose we approximate a smooth function f at x_n by a quadratic rather than a linear approximation: $f(\xi) = f(x_n) + (\xi - x_n)f'(x_n) + \frac{1}{2}(\xi - x_n)^2 f''(x_n)$ (see the textbook). Show that if x_n is close enough to the zero ξ , and if you pick the root of the quadratic equation closest to x_n , then the convergence is cubic, i.e. $|x_{n+1} - \xi| \leq C|x_n - \xi|^3$. Just do the asymptotic analysis, i.e. don't try to figure out exactly how close to ξ you have to start. What assumptions did you make about $f'(\xi)$ and $f''(\xi)$?

5. (due November 6th.) Write Matlab functions to implement the bisection method, secant method, the method of false position, and if you have time, Newton's method. It makes sense to implement these as a single function with the method fed in as a parameter. Your procedure should take as a parameter a function and an interval such that the function has different signs at the endpoints of the interval. It should stop when the zero is found to within a prescribed absolute tolerance. Test your procedure on the function $f(x) = \tan(\pi x) - 6$, with the initial interval $[0, 0.48]$. Note that the function has in this interval a root $\xi = (1/\pi) \arctan 6 \approx 0.447431543$. Run the procedure for 10, 20, and 50 iterations, and compare the results of the different methods. Discuss.