ORTHOGONAL POLYNOMIALS IN PROBABILITY THEORY
ABSTRACTS

MINI-COURSES

Jinho Baik (University of Michigan)
ORTHOGONAL POLYNOMIAL ENSEMBLES

TALK 1: ORTHOGONAL POLYNOMIAL ENSEMBLES

We introduce a certain joint density function of many particles on the line, given by the so-called orthogonal polynomials ensembles. Examples from random matrix theory and random tiling will be considered. Then we discuss the general theory of marginal distribution and gap probabilities of orthogonal polynomial ensembles.

TALK 2: BASICS OF ORTHOGONAL POLYNOMIALS

We now take a step back and consider a few basic properties of orthogonal polynomials such as three-term recurrence and connection to Hankel matrices. A so-called Riemann-Hilbert formulation of orthogonal polynomials will also be considered.

TALK 3: ASYMPTOTICS OF ORTHOGONAL POLYNOMIALS

The asymptotic result of orthogonal polynomials lead to an asymptotic result for orthogonal polynomial ensembles. We discuss such asymptotics through the Riemann-Hilbert formulation from Talk 2. Related variational problem will be discussed.

Roland Speicher (Queen's University, Canada)
FREE PROBABILITY AND FREE STOCHASTIC ANALYSIS

In this series of talks I will survey some of the basic notions, ideas, and results from free probability theory; with special emphasis on the occurrence of various orthogonal polynomials in that context. One main focus will be free stochastic analysis and the corresponding chaos decomposition of free Brownian motion. Also second order freeness and the diagonalization of fluctuations of random matrices will be treated.

TALKS

Alex Bloemendal (University of Toronto, Canada)
LIMITS OF SPIKED RANDOM MATRICES

Simple trends in high-dimensional data sets are modeled by spiked real Gaussian sample covariance matrices. In the large-size limit, the behavior of the top eigenvalue exhibits a phase transition as a function of the strength of the trend. Considering a tridiagonal form of the matrix as a discrete version of a certain random Schrödinger operator on the positive half-line, we show that the top eigenvalue has an asymptotic distribution near the phase transition; we give several characterizations of the limit laws, one of which involves only a linear boundary value problem. In the well-studied complex case, our PDE description reproduces known explicit formulas and yields a simple new proof of the Painlevé formula for the Tracy-Widom distribution. This is joint work with Bálint Virág.

Stephen Curran (University of California, Los Angeles)
GRAM MATRICES AND FREE QUANTUM GROUPS

There have been a number of recent results which suggest that in free probability, the roles of the classical permutation and orthogonal groups are played by Wang's universal compact quantum groups. In order to study the probabilistic features of these quantum groups, it is essential to have effective formulas for computing integrals with respect to their Haar measures. The Weingarten formula allows one to express these integrals in terms of the inverses of certain combinatorially defined Gram matrices. We will discuss some recent results about these matrices, and their probabilistic implications. We will also discuss the formula of Di Francesco, Golinelli and Guitter, which expresses the determinants of these Gram matrices as products of Chebyshev polynomials.

David Damanik (Rice University)
PSEUDO-RANDOMNESS AND ORTHOGONAL POLYNOMIALS

We start by recalling classical results on the asymptotics of orthogonal polynomials with random recursion coefficients and proceed to discuss how much the randomness may be reduced without losing the signature of randomness on the level of the orthogonal polynomials.

Holger Dette (Ruhr-Universität Bochum)
SOME ASYMPTOTIC PROPERTIES OF THE SPECTRUM OF THE JACOBI ENSEMBLE

For the random eigenvalues with density corresponding to the Jacobi ensemble

$$c \cdot \prod_{i<j} \mid \lambda_i - \lambda_j \mid^\beta \prod_{i=1}^{n} (2-\lambda_i)^a (2+\lambda_i)^b I_{(-2,2)}(\lambda_i)$$

$(a, b > -1, \beta > 0)$ a strong uniform approximation by the roots of the Jacobi polynomials is derived if the parameters a, b, β depend on n and $n \to \infty$.

Roughly speaking, the eigenvalues can be uniformly approximated by roots of Jacobi polynomials with parameters $((2a+2)/\beta-1, (2b+2)/\beta-1)$, where the error is of order $\{\log n/(a+b)\}^{1/4}$. These results are used to investigate the asymptotic properties of the corresponding spectral distribution if $n \to \infty$ and the parameters a, b and β vary with n. We also discuss further applications in the context of multivariate random F-matrices and extend these results to random block matrices.

This is joint work with Jan Nagel and Matthias Guhlich.

Maurice Duits (California Institute of Technology)
UNIVERSALITY IN THE TWO-MATRIX MODEL WITH A QUARTIC POTENTIAL

The eigenvalue statistics in the two-matrix model are described by certain biorthogonal polynomials. An asymptotic analysis for these polynomials leads to a description for the asymptotic behavior of the eigenvalues. In particular, it would answer the universality conjecture. However, for the general case such an asymptotic analysis is still an open problem. In this talk I will present some recent results on the special case where the potential for one of the matrices is quartic.

F. Alberto Grünbaum (University of California, Berkeley)
RANDOM WALKS AND ORTHOGONAL POLYNOMIALS: THE CLASSICAL AND THE QUANTUM CASE

I will review briefly the classical approach to Birth and death processes and orthogonal polynomials pushed mainly by S. Karlin and J. McGregor, some recent extensions involving matrix valued orthogonal polynomials and then show how these ideas have been recently adapted to the case of Quantum Random Walks in joint work with MJ Cantero, L. Moral and L. Velazquez. See “Matrix valued Szego polynomials and quantum random walks”, CPAM, vol 53, pages 464–507 (2010). I will try to emphasize some differences between the classical and the quantum case.

Yevgeniy Kovchegov (Oregon State University)
ORTHOGONAL POLYNOMIALS AND MIXING RATES

In this talk I will present an example of bounding total variation distance to stationarity and estimating mixing times via orthogonal polynomials diagonalization of discrete reversible Markov chains, the Karlin-McGregor approach. Next, I will compare the orthogonal polynomials approach to the other known techniques, such as geometric convergence and coupling. I will proceed by suggesting a method for estimating mixing rates for certain examples of reversible Markov chains over general state spaces.

Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium)
MULTIPLE ORTHOGONAL POLYNOMIALS IN RANDOM MATRIX THEORY

Multiple orthogonal polynomials (MOPs) are a generalization of orthogonal polynomials where the orthogonality is distributed among more than one orthogonality weight. MOPs are closely related to certain determinantal point process in the same way that usual orthogonal polynomials are related to eigenvalues of unitary invariant random matrix ensembles. A useful fact is that the correlation kernel in a MOP ensemble is expressed in terms of the solution of a Riemann-Hilbert problem which is of size 3×3 or larger. The unitary matrix model with external source is an example of a MOP ensemble which will be discussed in some detail for the case of a quartic potential. The large n-behavior is described by a vector equilibrium problem with external fields and a constraint. The model exhibits three kinds of phase transitions.

James A. Mingo (Queen's University, Canada)
ON THE FLUCTUATIONS OF THE KESTEN-MCKAY LAW

For $s > 1$, the Kesten-McKay Law describes the behaviour describes the the symmetric random walks on the free group with s generators as well as the limiting eigenvalue distribution of the adjacency matrix of a s-regular random graph.

We can define the fluctuations of this distribution by regarding it as limiting distributions of the sum of s independent Haar distributed random unitary matrices and their adjoints. We show that fluctuations can be diagonalized by some simple orthogonal polynomials.

This is joint work with Craig Armstrong, Roland Speicher, and Jenny Wilson.

Takuho Miyamoto (Tohoku University, Japan)
FREE ENTROPY DIMENSION OF PROJECTIONS AND FACTORIALITY OF GENERATED VON NEUMANN ALGEBRAS

We examine the free entropy and free entropy dimension for projections, and obtain a sufficient condition for the factoriality of the von Neumann algebra generated by projections in terms of their free entropy dimension. This corresponds to Voiculescu's result for self-adjoint elements.

Syeda Rabab Mudakkar (University of Nottingham, UK)
RADEMACHER INEQUALITY WITH APPLICATIONS

This work is motivated by optimal bounds in Rosenthal and Khintchine type moment inequalities. We establish several comparison results for commutative and non-commutative random variables including random matrices and freely independent variables. This is joint work with Dr. Sergey Utev.

Ivan Nourdin (Université Pierre et Marie Curie, France)
CUMULANTS ON THE WIENER SPACE

I will explain how to combine in finite-dimensional integration by parts procedures with a recursive relation on moments, in order to deduce explicit expressions for cumulants of functionals of a general Gaussian field. These findings yield a compact formula for cumulants on a fixed Wiener chaos, virtually replacing the usual "graph/diagram computations" adopted in most of the probabilistic literature. It is a joint work with Giovanni Peccati (Université du Luxembourg).

Victor Pérez-Abreu (CIMAT, Guanajuato, Mexico) CANCELED
SOME ROLES OF THE ARCSINE DISTRIBUTION IN INFINITE DIVISIBILITY

The arcsine distribution is not infinitely divisible with respect to classical and free convolutions. However, it plays an important role in the construction of infinitely divisible distribution in both classical and free senses. In this talk we present a survey of this role of the arcsine distribution including old and recent results.

Josep Lluís Solé (Universitat Autónoma de Barcelona, Spain)
THREE FAMILIES OF POLYNOMIALS RELATED TO A LÉVY PROCESS

We will study three different families of polynomials related to a Lévy process X. Concretely, we will deal with the Kaylath-Segall, Teugels and the time-space polynomials. The first family gives the relationship between the iterated integrals and the variations of order n of X. The second is the sequence (finite or infinite) of orthogonal polynomials $p_n^\sigma(x)$ with respect to the measure $\sigma^2\delta_0(dx) + x^2\,\nu(dx)$, where σ^2 is the variance of the Gaussian part of X and ν its Lévy measure. The latter family are the polynomials in space and time $Q(x,t)$, such that the process $M_t = Q(X_t,t)$ is a martingale with respect to the filtration associated to X. We will present some results as a characterization of the Lévy processes such that the Kaylath-Segall polynomials depend on a fixed number of variables, and an approximation of X with a sequence of simple Lvy processes $\{X_k,\ k \geq 1\}$ that converges in the Skorohod topology to X, and such that all variations and iterated integrals of X_k converge to the variations and iterated integral of X.

Frederic Utzet (Universitat Autónoma de Barcelona, Spain)
MULTIPLE STRATONOVICH INTEGRAL AND HU–MEYER FORMULA FOR LÉVY PROCESSES

In the framework of vector measures and the combinatorial approach to stochastic multiple integral introduced by Rota and Wallstrom [*Stochastic integrals: A combinatorial approach*, The Annals of Probability, **25** (1997) 1257–1283], we will study an Itô multiple integral and a Stratonovich multiple integral with respect to a Lévy process with finite moments up to a convenient order. In such a framework, the Stratonovich multiple integral is an integral with respect to a product random measure whereas the Itô multiple integral corresponds to integration with respect to a random measure that gives zero mass to the diagonal sets. A general Hu–Meyer formula that gives the relationship between both integrals will be proved. As particular cases, the classical Hu–Meyer formulas for the Brownian motion and for the Poisson process will be deduced. Furthermore, a pathwise interpretation for the multiple integrals with respect to a subordinator will be given.

This is joint work with Mercè Farré and Maria Jolis, Department of Mathematics, Universitat Autònoma de Barcelona.

Dong Wang (University of Michigan) CANCELED
HERMITIAN MATRIX MODEL WITH SPIKED EXTERNAL SOURCE.

The Hermitian matrix model is usually analyzed by a Riemann-Hilbert problem of size higher than 2. If the external source is spiked, i.e., only finitely many eigenvalues of the external source matrix are nonzero, we show a new approach to solve the problem. First we solve the rank 1 case by steepest-descent method, and then by a determinantal formula we derive the result in the higher rank case from that in the rank 1 case. We show the asymptotic behavior of the largest eigenvalue. This is joint work with Jinho Baik.