

Math 209A, Homework 3
due October 14

1. Royden, 2.42.

2. A real-valued function f is *lower semicontinuous* if for any λ , the set $\{x : f(x) > \lambda\}$ is open. What can you say about the sets $\{x : f(x) > \alpha\}$, $\{x : f(x) \geq \alpha\}$, $\{x : f(x) < \alpha\}$, $\{x : f(x) \leq \alpha\}$, and $\{x : f(x) = \alpha\}$?

3. Royden 3.4.

4. Royden 3.10.

5. Prove the *first Borel-Cantelli lemma*: If $\langle E_k \rangle_{k=1}^{\infty}$ is a sequence of sets with

$$\sum_{k=1}^{\infty} mE_k < \infty,$$

then $m(\limsup E_k) = 0$.