

**Math 209A, Homework 7**  
due December 2

1.

(a) Use Jensen's inequality to prove the following generalization of Young's inequality:

$$a_1 a_2 \dots a_n \leq \sum_{i=1}^n \frac{a_i^{p_i}}{p_i},$$

where  $a_i \geq 0$ ,  $p_i \geq 1$ , and  $\sum_{i=1}^n (1/p_i) = 1$ .

*Hint:* write  $a_i = \exp(x_i/p_i)$  and use the convexity of  $e^x$ .

(b) Prove the following generalization of Hölder's inequality: for  $\sum_{i=1}^n \frac{1}{p_i} = 1$ ,  $p_i \geq 0$ ,

$$\|f_1 f_2 \dots f_n\|_1 \leq \|f_1\|_{p_1} \dots \|f_n\|_{p_n}.$$

There are at least two ways to do this: use part (a), or the generalized Hölder inequality done in class.

2. Royden, Problem 6.2.

3. Royden, Problem 6.7.

4. Royden, Problem 6.13.

In the next two problem, one method is to use Theorem 4.17.

5. Royden, Problem 6.16.

6. Royden, Problem 6.17.