EDUCATIONAL CONCENTRATION WEEK IN FREE PROBABILITY ABSTRACTS

INTRODUCTION.

The two introductory talks will begin with the basics: definitions and some examples of freeness, free convolutions and free products. These topics will be used in the lecture series throughout the week. Some further topics and directions in free probability theory will be outlined, without going into details.

Hari Bercovici (Indiana University).

Lecture 1: THE MOMENT PROBLEM. Items to be discussed would be the relationship between moments, generating functions, and selfadjoint operators. The focus would be on the analysis required to recover a measure from its moments or Cauchy transform. An important item would be the invertibility of Cauchy transforms, particularly for measures concentrated around the origin. (Background required: basic measure theory, complex analysis and operator theory)

Lecture 2: FREE CONVOLUTIONS AND CAUCHY TRANSFORMS. This lecture would contain the Voiculescu-Haagerup proof for the additive convolution, but just state the multiplicative version for lack of time. (Background: free convolution and the Fock space model)

Lecture 3: LIMIT THEOREMS OF FREE PROBABILITY THEORY. Here we would actually prove that convergence of triangular arrays is infinitely divisible, as well as conditions for convergence. A discussion of Levy-Hincin formulas would have to be included. (Background: nothing additional really)

Lecture 4: FINER PROPERTIES OF FREE CONVOLUTION. The theme is subordination (with a possibly complete argument) and how it is used to prove regularity properties of free convolutions, such as absolute continuity. Here we might restrict to convolution powers where the arguments are more direct.

Alexandru Nica (University of Waterloo), LECTURE SERIES ON THE COMBINATORICS OF FREE PROBABILITY THEORY.

In these lectures we will look at lattices of non-crossing partitions and at some quantities called free cumulants (for a family of non-commutative random variables) which are defined by using non-crossing partitions. We will see how free independence is expressed in terms of free cumulants, and how this can be used in order to efficiently perform certain calculations involving free random variables. The emphasis of the lectures will be on "concrete" examples illustrating how problems about free random variables can be reduced to statements about non-crossing partitions, and thus solved via combinatorial arguments. The material presented will be drawn from the recent monograph "Lectures on the combinatorics of free probability" by A. Nica and R. Speicher (Cambridge University Press, 2006).

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Specific topics for the four lectures:

Lecture 1. Non-crossing partitions and free cumulants
Lecture 2. Examples of calculations with free cumulants I
Lecture 3. Examples of calculations with free cumulants II
Lecture 4. Free cumulants and asymptotics for GUE and Wishart random matrices.

Steen Thorbjørnsen (University of Aarhus)

Synopsis of Talk no. 1. Random matrices and their spectral distributions, The Gaussian Unitary Ensemble, Wigner's Semi-circle Law, The Harer-Zagier Recursion Formulae, asymptotics for the largest and smallest eigenvalue of GUE random matrices.

Synopsis of talk no. 2. Mixed moments of independent GUE random matrices, Voiculescu's random matrix model for a free semi-circular system, Dykema's generalization, applications to the free group von Neumann factors.

Synopsis of talk no. 3. Applications of random matrices to the free group von Neumann factors (continued), convergence of norms in Voiculescu's random matrix model, asymptotic expansion for C_c^∞-functions of a polynomial of independent GUE random matrices, Capitaine's and Donati-Martin's generalization.

Synopsis of talk no. 4. Applications of random matrices to the reduced C^*-algebra associated to the free group on two (or more) generators: The extension semi-group is not a group and there are no projections, applications of random matrices to general exact C^*-algebras.

Benoît Collins (University of Ottawa and Université Lyon 1), A LINEARIZATION THEOREM FOR NON-COMMUTATIVE PROBABILITY SPACES AND APPLICATIONS TO CONNES EMBEDDABILITY CONJECTURE.

We prove a von Neumann non-commutative probability space analogue of a C^*-algebra linearization trick by Haagerup-Thorbjørnsen. We apply it to derive a new equivalent statement of Connes embeddability conjecture. It is in terms of possible spectral measures of sum of some selfadjoint elements with matrix coefficients.

This is joint work with Ken Dykema.

Nizar Demni (Université Paris 6-7), FREE JACOBI PROCESSES.

In this paper, we define and study two parameters dependent free processes (λ, θ) called *free Jacobi*, obtained as the limit of its matrix counterpart when the size of the matrix goes to infinity. The main result we derive is a free SDE analogous to that satisfied in the matrix setting, derived under injectivity assumptions. Once we did, we examine a particular case for which the spectral measure is explicit and does not depend on time (stationary). This allows us to determine easily the parameters range ensuring our injectivity requirements so that our result applies. Then, we show that under an additional condition of invertibility at time $t = 0$, this range extends to the general setting. To proceed, we set a recurrence formula for the moments of the process via free stochastic calculus.

Nermine El-Sissi (Trinity University), POSITIVE DEFINITE KERNELS AND LATTICE PATHS.

We discuss the structure of positive definite kernels in terms of operator models. We introduce two models, one of Hessenberg type and another we call near tridiagonal. These models produce

parametrization of the kernels. We also discuss their combinatorial nature in terms of lattice paths of Dyck and Lukasiewicz type.

Takuho Miyamoto (Tohoku University), ORBITAL APPROACH TO MICROSTATE FREE ENTROPY.

Motivated by Voiculescu's liberation theory, we introduce the orbital free entropy χ_{orb} for non-commutative self-adjoint random variables (also for "hyperfinite random multivariables"). Besides its basic properties the relation of χ_{orb} with the usual free entropy χ is shown. Moreover, the dimension counterpart of χ_{orb} is discussed.

Suat Namli (Louisiana State University), MULTIPLICATIVE RENORMALIZATION METHOD FOR ORTHOGONAL POLYNOMIALS.

We demonstrate an alternative to the classical Gram-Schmidt process to find the orthogonal polynomials for a given measure, with motivation from infinite dimensional white noise analysis. Instead of finding the orthogonal polynomials recursively as described in the Gram-Schmidt process, we analyze different types of generating functions systematically and come up with polynomials after power expansion. This work also produces the Jacobi-Szego parameters easily and hence one can work on the one-mode Interacting Fock Space related to these parameters. We have verified the classical measures and corresponding orthogonal polynomials and we found some new measures which generalize the Wigner distributions.

Anthony Reveillac (Universite de La Rochelle), STEIN ESTIMATION USING MALLIAVIN CALCULUS.

In this talk we will review the construction of Malliavin calculus for the free Brownian motion on the one hand, and some recent applications of classical (commutative) Malliavin calculus to transportation inequalities and to statistical estimation on the other hand. Our goal will be to expose the possible connections between these two topics.

Junhao Shen (University of New Hampshire), TOPOLOGICAL FREE ENTROPY DIMENSION IN UNITAL C^* ALGEBRAS.

The notion of topological free entropy dimension was introduced by D. Voiculescu. During the talk, we will first compute the topological free entropy dimension of one self-adjoint element in a unital C^* algebra. Then we will discuss a few properties of topological free entropy dimension, for example: the topological free entropy dimension in the orthogonal sums of unital C^* algebra; the topological free entropy dimension in the tensor product of a unital C^*algebra with an n by n matrix algebra. In the last, we will discuss the topological free entropy dimensions in a unital, simple, infinite dimensional, C^* algebra with a unique trace.