Math 308 plots

February 3, 2009.

The three plots on this page are of the external temperature

$$M(t) = 48 - 13\cos(\omega t),$$

internal temperature

$$T(t) = 54 - 10.2\cos(\omega t - 0.5) + 11.5e^{-t/2},$$

and the oscillatory (long-term) component of the internal temperature

$$T_{\text{osc}}(t) = 54 - 10.2\cos(\omega t - 0.5).$$

The governing ODE is

$$\frac{dT}{dt} = K(M(t) - T(t)) + H_0,$$

where

$$M(t) = M_0 - B\cos(\omega t)$$

and the time constant $1/K = 2$ hours.

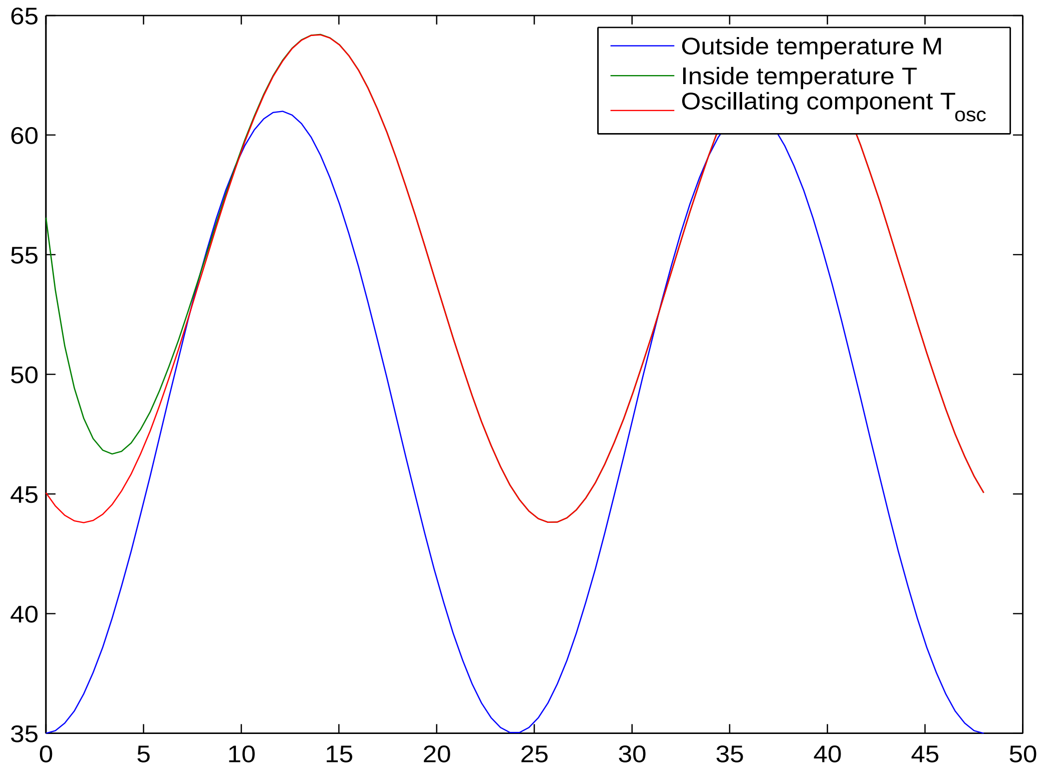

FIGURE 1. Temperature as a function of time with constant heating.

The three plots on this page are of the external temperature

$$M(t) = 48 - 13\cos(\omega t),$$

internal temperature

$$T(t) = \frac{1}{5}\cdot 54 + \frac{4}{5}\cdot 65 - \frac{1}{5}\cdot 10.2\cos(\omega t - 0.5) + 11.5e^{-2.5t},$$

and the oscillatory (long-term) component of the internal temperature

$$T_{\text{osc}}(t) = \frac{1}{5}\cdot 54 + \frac{4}{5}\cdot 65 - \frac{1}{5}\cdot 10.2\cos(\omega t - 0.5).$$

The governing ODE is

$$\frac{dT}{dt} = K(M(t) - T(t)) + K_U(T_D - T(t)),$$

where

$$M(t) = M_0 - B\cos(\omega t)$$

and the time constant for the building with heating $1/K_U = 1/2$ hour.

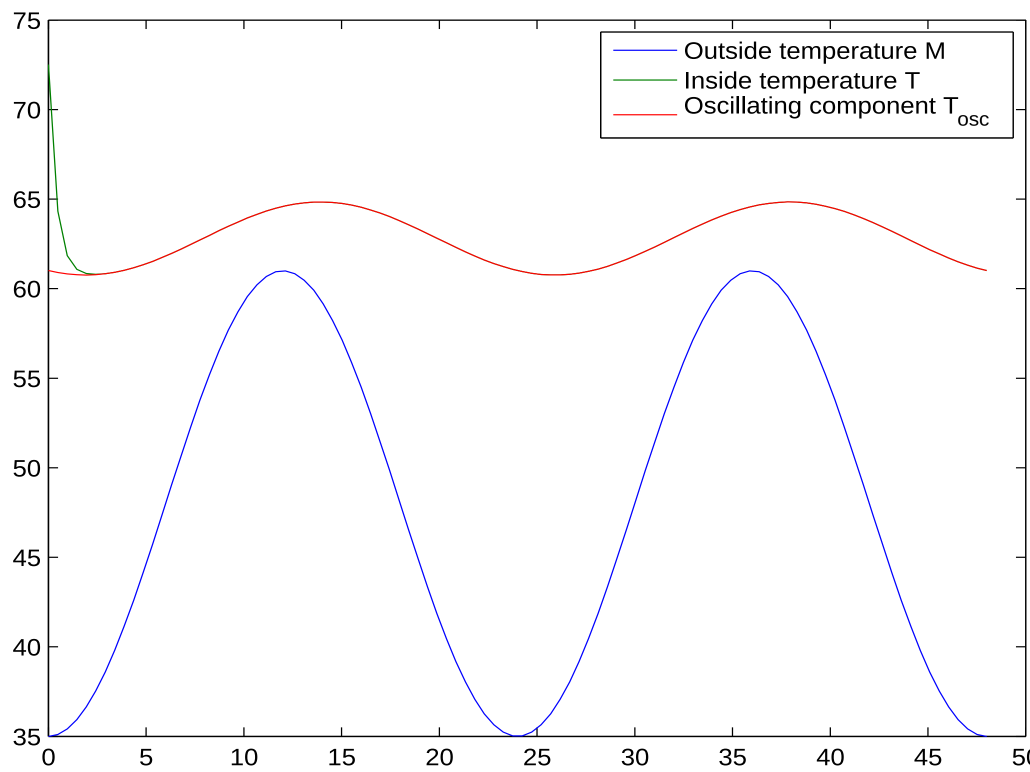

FIGURE 2. Temperature as a function of time with heating proportional to the temperature difference.

Solution to the Exercise 3.3.16. We want to re-write $C_1 \cos \omega t + C_2 \sin \omega t$ as a single cosine curve. Let

$$A = \sqrt{C_1^2 + C_2^2} \qquad \text{(amplitude)},$$
$$\phi = \arctan(C_2/C_1) \qquad \text{(phase)},$$

so that

$$\tan \phi = C_2/C_1, \quad \cos \phi = \frac{C_1}{\sqrt{C_1^2 + C_2^2}} = \frac{C_1}{A}, \quad \sin \phi = \frac{C_2}{\sqrt{C_1^2 + C_2^2}} = \frac{C_2}{A}$$

(draw a triangle). Then using the trig formula

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta,$$

we get

$$A \cos(\omega t - \phi) = A \cos \phi \cos \omega t + A \sin \phi \sin \omega t = C_1 \cos \omega t + C_2 \sin \omega t$$

as desired.