Math 308 practice midterm exam II

SELECTED LAPLACE TRANSFORMS

f	1	t^n	e^{at}	$\sin bt$	$\cos bt$	$u(t-a)$	$\delta(t-a)$
$\mathcal{L}[f]$	$\frac{1}{s}$	$\frac{n!}{s^{n+1}}$	$\frac{1}{s-a}$	$\frac{b}{s^2+b^2}$	$\frac{s}{s^2+b^2}$	$e^{-as}\frac{1}{s}$	e^{-as}

1. Solve the initial value problem

$$y'' + 9y = e^{3x}, \qquad y(0) = 0, \quad y'(0) = 1.$$

2. Determine the form of a particular solution for the following equation. You do **not** have to find the solution itself.

$$y'' - 4y' + 4y = \sin x + \cos 2x + e^{2x} + 2x.$$

3. Consider the differential equation

$$y'' - \frac{1}{x}y' + \frac{1}{x^2}y = 0.$$

(a) Verify that the functions

$$y_1 = x, \qquad y_2 = x \ln x$$

are solutions of this differential equation.

(b) Compute the Wronskian of y_1 and y_2, and use this to tell if these two functions form a fundamental set of solutions for $x > 0$ (which is where y_2 is defined).

(c) The Variation of Parameters method states that if the equation is in standard form with the right-hand-side F, then

$$v_1(x)y_1(x) + v_2(x)y_2(x)$$

is a solution to it for

$$v_1'(x) = -\frac{F(x)y_2(x)}{W[y_1, y_2](x)}, \qquad v_2'(x) = \frac{F(x)y_1(x)}{W[y_1, y_2](x)}.$$

Use this information to find a particular solution of the equation

$$y'' - \frac{1}{x}y' + \frac{1}{x^2}y = \frac{1}{x}.$$

4.

(a) Compute the Laplace transform

$$\mathcal{L}[t^2 e^{-2t}].$$

(b) Compute the inverse Laplace transform

$$\mathcal{L}^{-1}\left[\frac{1}{(2s-1)^3}\right].$$

5. Consider the non-linear system

$$\frac{dx}{dt} = 16 - xy$$
$$\frac{dy}{dt} = x - y^3.$$

(a) Find all the critical points of the system.
(b) For one of these critical points (make sure to state which one), determine its type.