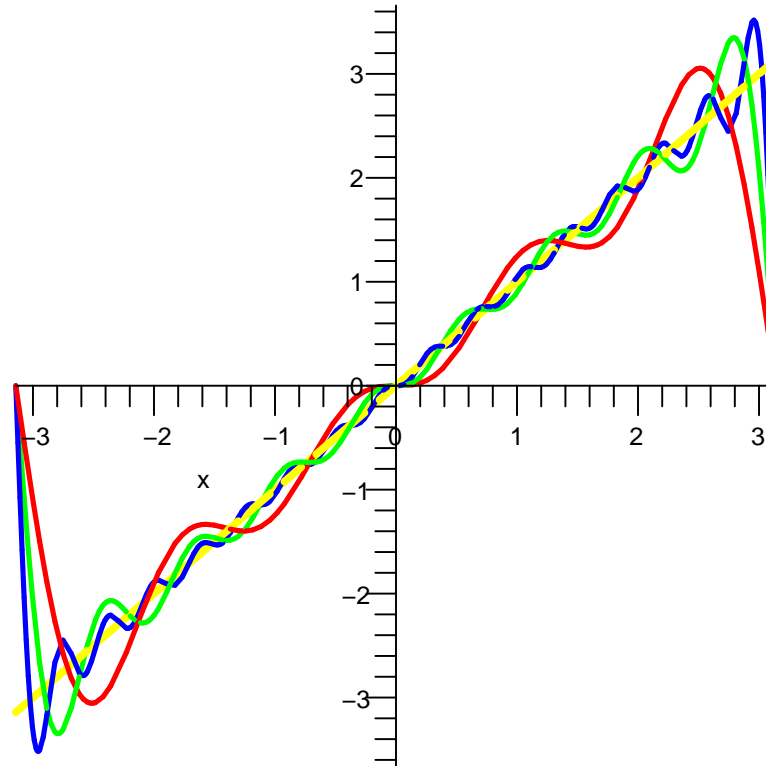


Math 311 handout on Fourier series

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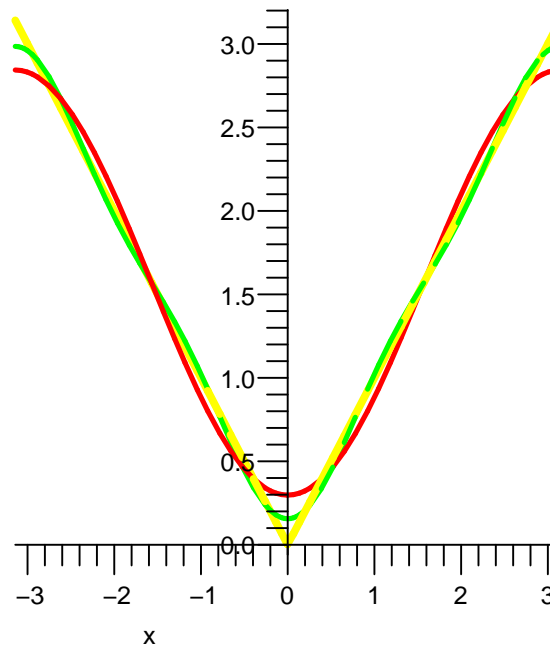
March 14, 2008

$$f(x) = x. \text{ Fourier series } S_N(x) = -2 \sum_{k=1}^N \frac{(-1)^k}{k} \sin(kx).$$



Using 4, 8, 16 terms on $[-\pi, \pi]$. Approximates well, except at the endpoints: values 0 instead of $\pm\pi$.

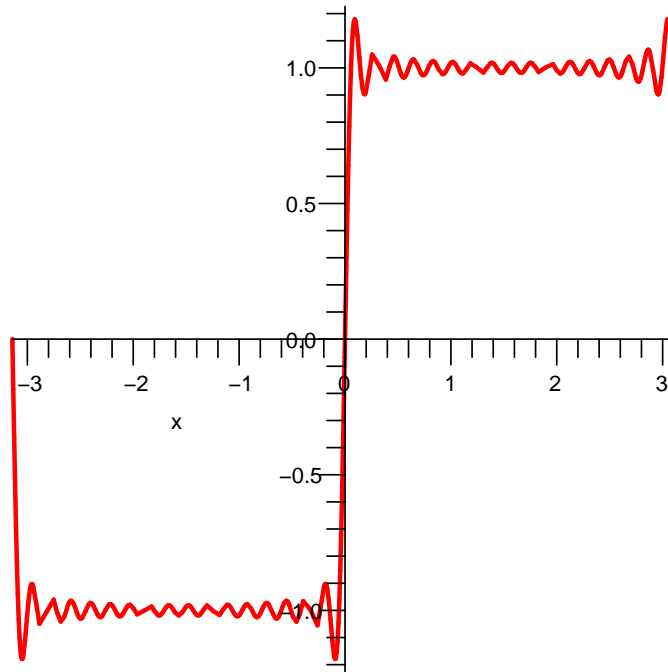
$$f(x) = |x|. \text{ Fourier series } U_N(x) = \frac{1}{2}\pi - \frac{4}{\pi} \sum_{k=0}^N \frac{1}{(2k+1)^2} \cos((2k+1)x).$$



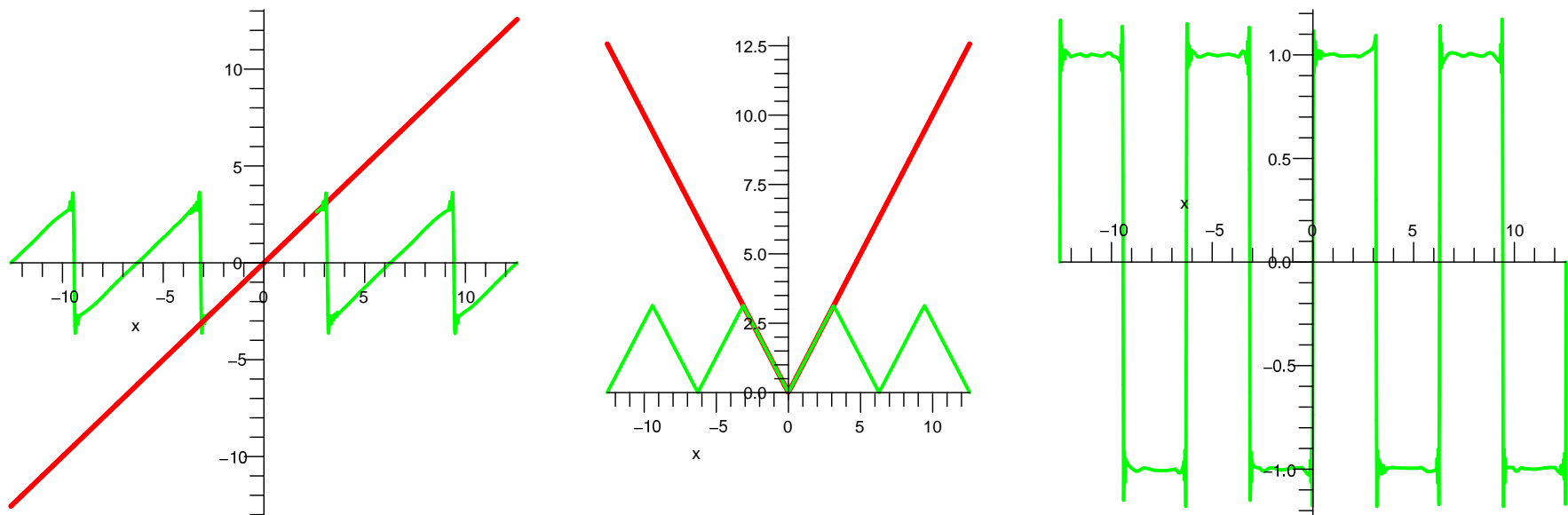
With 0, 1 terms. Why are so few terms enough?

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0, \\ 1, & 0 \leq x \leq \pi. \end{cases} \text{ Fourier series}$$

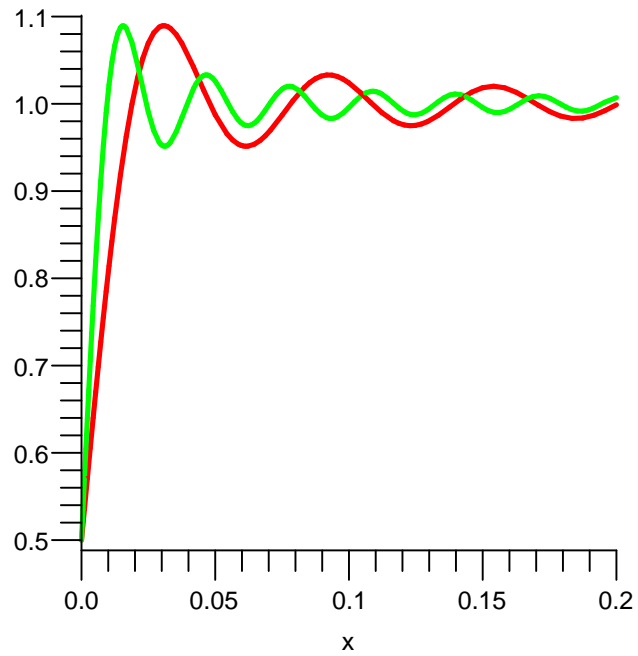
$$T_N(x) = \frac{4}{\pi} \sum_{k=0}^N \frac{1}{2k+1} \sin((2k+1)x).$$



Approximation on the whole real line, with 50 terms. Note the end-point jumps (on the first but not the second graph) and periodicity. Also note the Gibbs phenomenon.



Gibbs phenomenon does not disappear for better approximations: for 50 vs. 100 terms



Pointwise but not uniform convergence.