AIRY FUNCTIONS.

Solutions of the equation

$$y''(x) - xy(x) = 0$$

(note sign).

$$\mathrm{Ai}(x) = \frac{1}{3}\frac{\sqrt[3]{3}}{\Gamma(2/3)} - \frac{1}{2}\frac{\sqrt[6]{3}\Gamma(2/3)}{\pi}x + \frac{1}{18}\frac{\sqrt[3]{3}}{\Gamma(2/3)}x^3 - \frac{1}{24}\frac{\sqrt[6]{3}\Gamma(2/3)}{\pi}x^4 + \frac{1}{540}\frac{\sqrt[3]{3}}{\Gamma(2/3)}x^6 - \frac{1}{1008}\frac{\sqrt[6]{3}\Gamma(2/3)}{\pi}x^7 + \frac{1}{38880}\frac{\sqrt[3]{3}}{\Gamma(2/3)}x^9 + O\left(x^{10}\right)$$

and

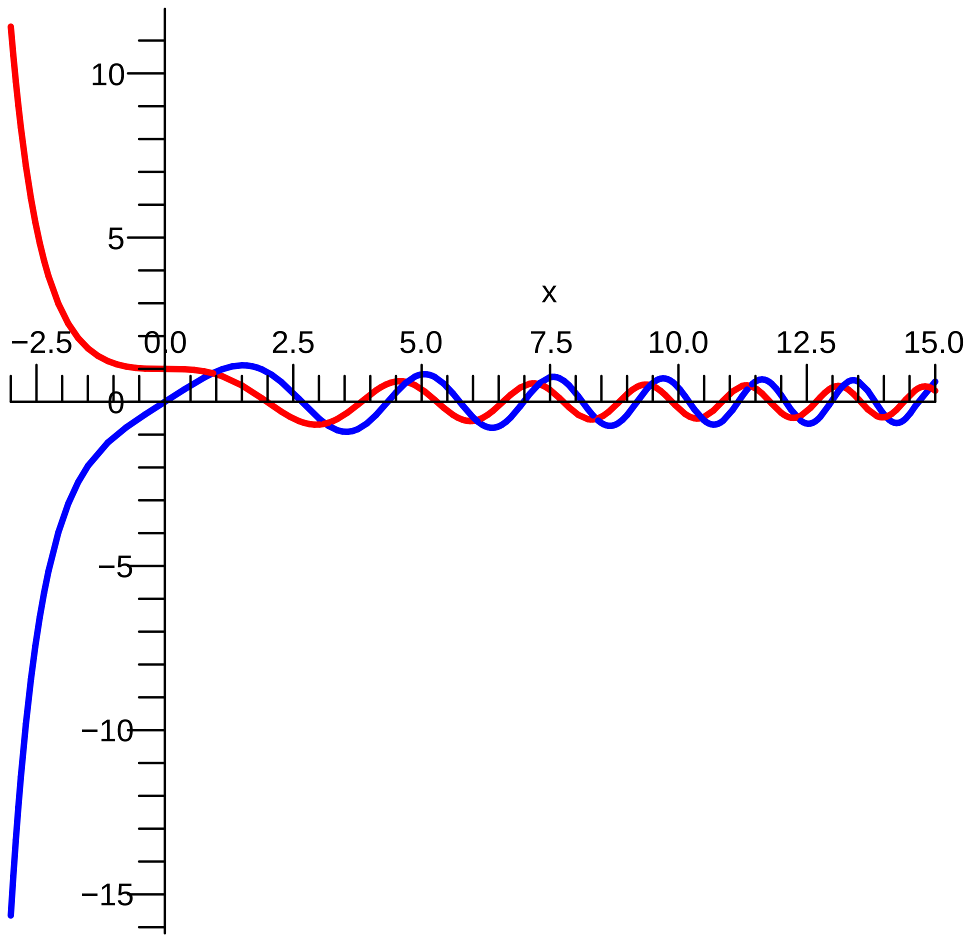
$$\mathrm{Bi}(x) = \frac{1}{3}\frac{3^{5/6}}{\Gamma(2/3)} + \frac{1}{2}\frac{3^{2/3}\Gamma(2/3)}{\pi}x + \frac{1}{18}\frac{3^{5/6}}{\Gamma(2/3)}x^3 + \frac{1}{24}\frac{3^{2/3}\Gamma(2/3)}{\pi}x^4 + \frac{1}{540}\frac{3^{5/6}}{\Gamma(2/3)}x^6 + \frac{1}{1008}\frac{3^{2/3}\Gamma(2/3)}{\pi}x^7 + \frac{1}{38880}\frac{3^{5/6}}{\Gamma(2/3)}x^9 + O\left(x^{10}\right)$$

Our Airy functions: solutions of $y'' + xy = 0$, with initial conditions $y(0) = 1$, $y'(0) = 0$ and $y(0) = 0$, $y'(0) = 1$.

$$\frac{1}{2}\Gamma(2/3)3^{1/6}\left(\sqrt{3}\,\mathrm{Ai}(-x) + \mathrm{Bi}(-x)\right)$$
$$= 1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \frac{1}{12960}x^9 + \frac{1}{1710720}x^{12} - \frac{1}{359251200}x^{15}$$
$$+ \frac{1}{109930867200}x^{18} - \frac{1}{46170964224000}x^{21} + O(x^{24}).$$

$$-\frac{1}{3}\frac{\pi 3^{1/3}}{\Gamma(2/3)}\left(\mathrm{Bi}(-x) - \sqrt{3}\,\mathrm{Ai}(-x)\right) = x - \frac{1}{12}x^4 + \frac{1}{504}x^7 + O(x^{10}).$$

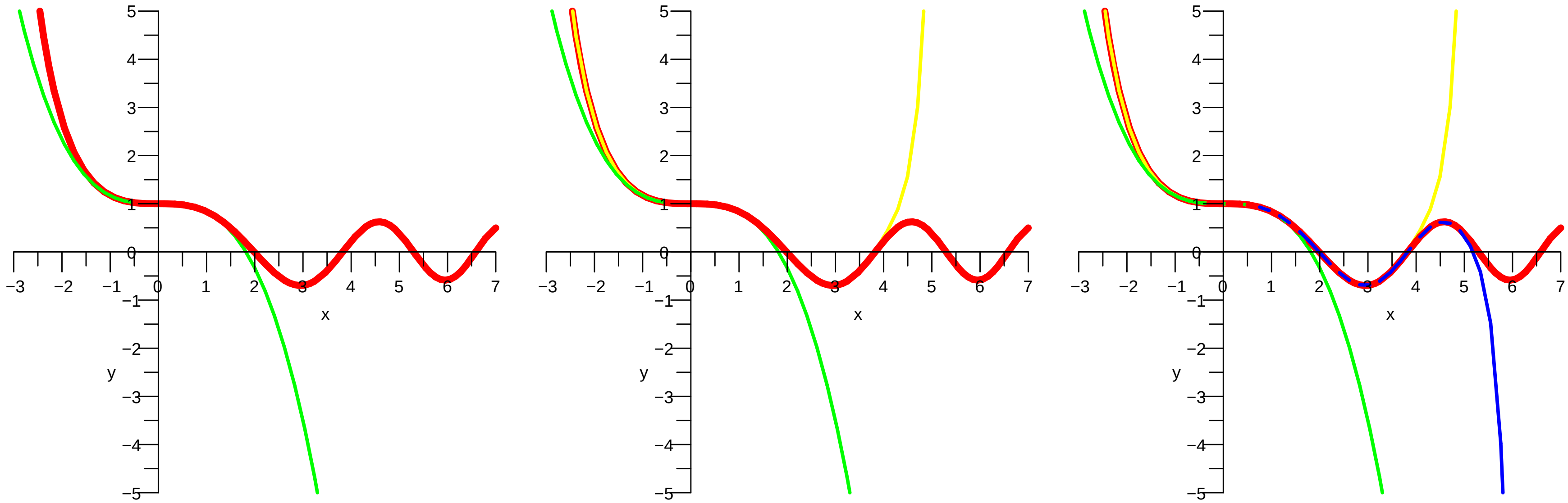
Airy functions.



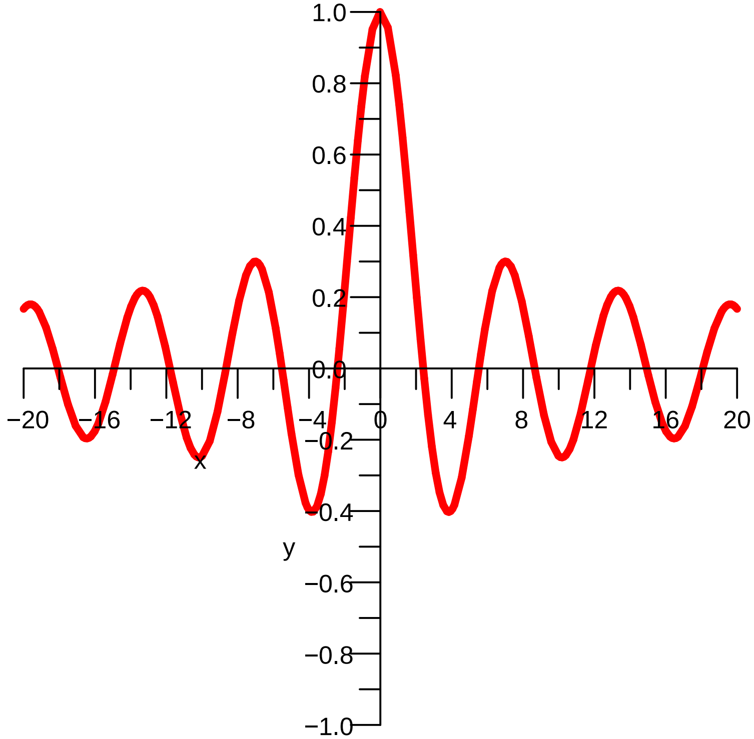
Approximations by

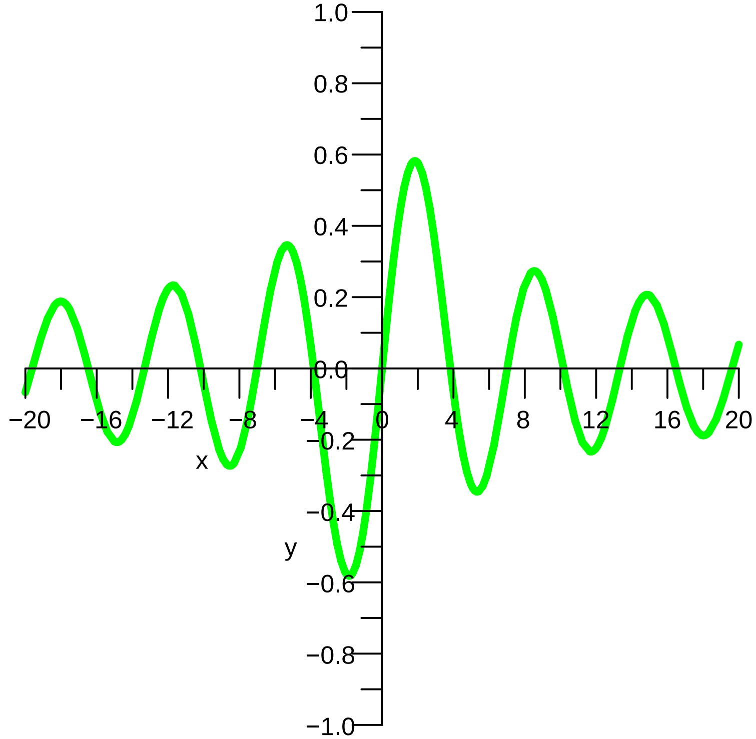
$$1-\frac{1}{6}x^3+\frac{1}{180}x^6-\frac{1}{12960}x^9+\frac{1}{1710720}x^{12}-\frac{1}{359251200}x^{15}+O(x^{18})$$

of order 3, 18, and 27.

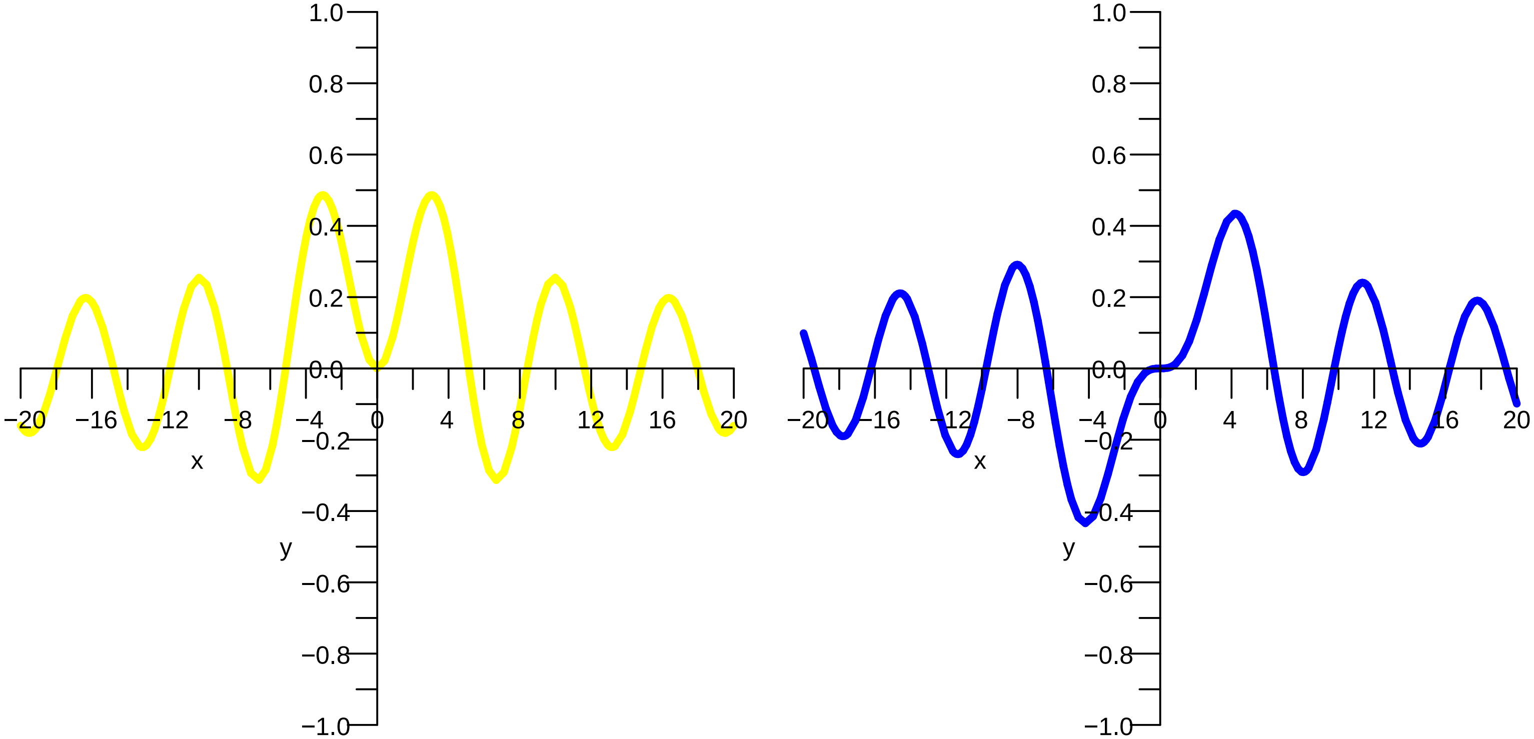


Bessel functions J_0 and J_1.





Bessel functions J_2 and J_3.



First four Bessel functions.