

Math 409, Homework 4
due March 1

Section 500.

2.3. 1, 3(a), 4.

2.4. 7.

3.1. 1(a,b), 2, 3(a-c), 4.

Section 200.

2.3. 3(a), 4.

2.4. 7.

3.1. 1(a,b), 2, 3(a-c).

Problem 7.

(a) Prove that for any $x, a \in \mathbb{R}$ and $n \in \mathbb{N}$,

$$x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + xa^{n-2} + a^{n-1}) = (x - a) \sum_{i=0}^{n-1} x^{n-1-i} a^i.$$

You can prove this by induction, or just multiply through, I am not asking for complete rigor.

(b) Prove that if $|x - a| < 1$ then

$$|x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1}| < (|a| + 1)^n - |a|^n.$$

(c) Use this result to prove directly from the definition that

$$\lim_{x \rightarrow a} x^n = a^n.$$

(d) Prove the result in part (c) using limit theorems. Note that you still have to calculate *some* limit directly from the definition, but possibly a much simpler one.

Problem 8. Suppose $L = \lim_{x \rightarrow a} f(x)$ exists and $L > 0$. Prove that there is a (small) open interval I containing a such that for all $x \in I \setminus \{a\}$, $f(x) > 0$.