

## Math 409, Homework 9

due April 19.

### Section 500.

**5.2.** 4, 5, 6(a).

**5.3.** 1(a-d), 3(a-b). Do not do 3(c-d).

**Problem 6.** Let  $f$  be continuous on  $[a, b]$ . for each  $x \in [a, b]$ , let  $F(x) = \int_x^b f(t) dt$ . Prove that  $F$  is differentiable and that

$$F'(x) = -f(x).$$

(You may want to do this problem before 5.3.3.)

**Problem 7.** Let

$$f(x) = \begin{cases} x, & 0 \leq x \leq 2, \\ 3, & 2 < x \leq 4. \end{cases}$$

- (a) Find an explicit expression for  $F(x) = \int_0^x f(t) dt$  as a function of  $x$ .
- (b) Sketch  $F$  and determine where  $F$  is differentiable.
- (c) Find a formula for  $F'(x)$  wherever  $F$  is differentiable.

**Problem 8.** If  $f$  is an integrable even function, prove that  $F(x) = \int_0^x f(t) dt$  is an odd function.

(If you don't remember the definitions of even and odd functions, you can look them up.)

**Section 200 (honors).**

**5.2.** 4, 5, 6(a).

**5.3.** 3(a-b), 4. Do not do 3(c-d).

**Problem 6.** Let  $f$  be continuous on  $[a, b]$ . for each  $x \in [a, b]$ , let  $F(x) = \int_x^b f(t) dt$ . Prove that  $F$  is differentiable and that

$$F'(x) = -f(x).$$

(You may want to do this problem before 5.3.3.)

**Problem 7.** Let

$$f(x) = \begin{cases} x, & 0 \leq x \leq 2, \\ 3, & 2 < x \leq 4. \end{cases}$$

- (a) Find an explicit expression for  $F(x) = \int_0^x f(t) dt$  as a function of  $x$ .
- (b) Sketch  $F$  and determine where  $F$  is differentiable.
- (c) Find a formula for  $F'(x)$  wherever  $F$  is differentiable.

**Problem 8.** Let  $f$  be a continuously differentiable function and  $f(0) = 0$ . Define the function  $g : \mathbb{R} \rightarrow \mathbb{R}$  by

$$g(x) = \int_0^1 f'(tx) dt.$$

Prove that  $f(x) = xg(x)$ .