Section 500.
5.2. 4, 5, 6(a).
5.3. 1(a-d), 3(a-b). Do not do 3(c-d).

Problem 6. Let $f$ be continuous on $[a, b]$. for each $x \in [a, b]$, let $F(x) = \int_x^b f(t) \, dt$. Prove that $F$ is differentiable and that
$$F'(x) = -f(x).$$
(You may want to do this problem before 5.3.3.)

Problem 7. Let
$$f(x) = \begin{cases} 
  x, & 0 \leq x \leq 2, \\
  3, & 2 < x \leq 4.
\end{cases}$$
(a) Find an explicit expression for $F(x) = \int_0^x f(t) \, dt$ as a function of $x$.
(b) Sketch $F$ and determine where $F$ is differentiable.
(c) Find a formula for $F'(x)$ wherever $F$ is differentiable.

Problem 8. If $f$ is an integrable even function, prove that $F(x) = \int_0^x f(t) \, dt$ is an odd function.
(If you don’t remember the definitions of even and odd functions, you can look them up.)
Section 200 (honors).

5.2. 4, 5, 6(a).

5.3. 3(a-b), 4. Do not do 3(c-d).

Problem 6. Let \( f \) be continuous on \([a, b]\). for each \( x \in [a, b] \), let \( F(x) = \int_x^b f(t) \, dt \). Prove that \( F \) is differentiable and that
\[
F'(x) = -f(x).
\]
(You may want to do this problem before 5.3.3.)

Problem 7. Let
\[
f(x) = \begin{cases} 
x, & 0 \leq x \leq 2, \\
3, & 2 < x \leq 4.
\end{cases}
\]
(a) Find an explicit expression for \( F(x) = \int_0^x f(t) \, dt \) as a function of \( x \).
(b) Sketch \( F \) and determine where \( F \) is differentiable.
(c) Find a formula for \( F'(x) \) wherever \( F \) is differentiable.

Problem 8. Let \( f \) be a continuously differentiable function and \( f(0) = 0 \). Define the function \( g : \mathbb{R} \to \mathbb{R} \) by
\[
g(x) = \int_0^1 f'(tx) \, dt.
\]
Prove that \( f(x) = xg(x) \).