Math 447, Homework 6.

1. Suppose \( g : D \to [-\infty, \infty] \) is measurable. Using \( 1/0 = \infty \), define \( 1/g : D \to (-\infty, \infty] \) in a natural way. Prove that it is measurable.

2. Exercise 17.25 (page 303).

3. Exercise 17.33 (page 305). You may use without proof that a function continuous a.e. is measurable (cf. Theorem 17.4) and the results in Exercise 32.

4. Exercise 17.36 (page 306).

Quiz 6. Let \( D \) be a measurable set with \( m(D) < \infty \). Suppose the family of measurable functions \( \{f_n : D \to \mathbb{R}\} \) is pointwise bounded: for each \( x \in D \), there exists an \( M_x < \infty \) such that \( |f_n(x)| \leq M_x \) for all \( n \). Show that, given \( \varepsilon > 0 \), there exists a closed set \( F \subset D \) with \( m(D \setminus F) < \varepsilon \) such that this family is uniformly bounded on \( F \): for some finite \( M \), \( |f_n(x)| \leq M \) for all \( n \) and all \( x \in F \).
1. Suppose $g : D \to [-\infty, \infty]$ is measurable. Using $1/0 = \infty$, define $1/g : D \to (-\infty, \infty]$ in a natural way. Prove that it is measurable.

2. Exercise 17.31 (page 305).

3. Exercise 17.33 (page 305). You may use without proof that a function continuous a.e. is measurable (cf. Theorem 17.4) and the results in Exercise 32.

4. Exercise 17.36 (page 306).

**Honors Quiz 6.** Let $D$ be a measurable set with $m(D) < \infty$. Suppose the family of measurable functions $\{f_n : D \to \mathbb{R}\}$ is pointwise bounded: for each $x \in D$, there exists an $M_x < \infty$ such that $|f_n(x)| \leq M_x$ for all $n$. Show that, given $\varepsilon > 0$, there exists a closed set $F \subset D$ with $m(D \setminus F) < \varepsilon$ such that this family is uniformly bounded on $F$: for some finite $M$, $|f_n(x)| \leq M$ for all $n$ and all $x \in F$. 