1. **E** Use the trigonometric substitution \( x = \frac{2}{3} \tan \theta, \ dx = \frac{2}{3} \sec^2 \theta \ d\theta \). Then
\[
\int \frac{x^2}{\sqrt{9x^2 + 4}} \ dx = \int \frac{\left( \frac{2}{3} \tan \theta \right)^2}{\sqrt{\frac{4}{9} \tan^2 \theta + 4}} \cdot \frac{2}{3} \sec^2 \theta \ d\theta
\]
\[
= \int \frac{\frac{4}{9} \tan^2 \theta \cdot 2}{2 \sec \theta} \cdot \frac{2}{3} \sec^2 \theta \ d\theta
\]
\[
= \frac{4}{27} \int \tan^2 \theta \sec \theta \ d\theta.
\]

2. **A**
\[
\int_1^\infty \frac{1}{e^{2x}} \ dx = \lim_{b \to \infty} \int_1^b e^{-2x} \ dx = \lim_{b \to \infty} \left[ -\frac{1}{2} e^{-2x} \right]_1^b = \lim_{b \to \infty} \left[ -\frac{1}{2e^{2b}} + \frac{1}{2e^2} \right] = 0 + \frac{1}{2e^2} = \frac{1}{2e^2}.
\]

3. **B** Because \( x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1) \), we see that the denominator has two distinct linear factors and one irreducible quadratic factor. Therefore, the partial fraction decomposition is
\[
\frac{1}{x^4 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 1}.
\]

4. **A** Splitting \([1, 3]\) into \( n = 6 \) equal subintervals, we find width of each subinterval is \( \frac{3 - 1}{6} = \frac{2}{3} \). Thus the end-points of the subintervals are
\[
x_0 = 1, \ x_1 = \frac{4}{3}, \ x_2 = \frac{5}{3}, \ x_3 = 2, \ x_4 = \frac{7}{3}, \ x_5 = \frac{8}{3}, \ x_6 = 3.
\]
Checking with the formula for the trapezoidal rule, we see that the answer must be A.

5. **C** We quickly evaluate the three integrals:
\[
\int_1^\infty \frac{1}{x} \ dx = \lim_{b \to \infty} \int_1^b \frac{1}{x} \ dx = \lim_{b \to \infty} \left[ \ln x \right]_1^b = \lim_{b \to \infty} \left[ \ln b - 0 \right] = \infty,
\]
\[
\int_{-\infty}^\infty \sin x \ dx = \lim_{a \to -\infty} \int_a^0 \sin x \ dx + \lim_{b \to \infty} \int_0^b \sin x \ dx
\]
\[
= \lim_{a \to -\infty} \left[ -\cos x \right]_a^0 + \lim_{b \to \infty} \left[ -\cos x \right]_0^b
\]
\[
= \lim_{a \to -\infty} \left[ -1 - \cos a \right] + \lim_{b \to \infty} \left[ \cos b + 1 \right] \quad \text{(neither limit exists)},
\]
\[
\int_0^1 \frac{1}{\sqrt{x}} \ dx = \lim_{a \to 0^+} \int_a^1 \frac{1}{\sqrt{x}} \ dx = \lim_{a \to 0^+} \left[ 2\sqrt{x} \right]_a^1 = \lim_{a \to 0^+} \left[ 2 - 2\sqrt{a} \right] = 2.
\]
Thus, I and II both diverge, and III converges.
6. To find the arc length of the curve, we use the arc length formula 
\[ L = \int_a^b \sqrt{(y')^2 + 1} \, dx. \]
We use 
\[ y = 2x^{3/2} \] and \([a, b] = [0, 2]\). Since 
\[ y' = 3x^{1/2}, \]
we have 
\[ L = \int_0^2 \sqrt{(3x^{1/2})^2 + 1} \, dx = \int_0^2 \sqrt{9x + 1} \, dx. \]
\[ = \int_1^{19} \frac{1}{9} \sqrt{u} \, du \quad \text{(using } u = 9x + 1, \, du = 9 \, dx) \]
\[ = \left[ \frac{2}{27} u^{3/2} \right]_1^{19} = \frac{2}{27} 19^{3/2} - \frac{2}{27}. \]

7. (a) For this integral, we first perform long division and find 
\[ \frac{x^4}{x^2 + 4} \, dx = x^2 - 4 + \frac{16}{x^2 + 4}, \]
so 
\[ \int \frac{x^4}{x^2 + 4} \, dx = \left( x^2 - 4 + \frac{16}{x^2 + 4} \right) \, dx \]
\[ = \frac{x^3}{3} - 4x + 16 \int \frac{1}{4 \sec^2 \theta} \cdot 2 \sec^2 \theta \, d \theta \quad \text{(using } x = 2 \tan \theta, \, dx = 2 \sec^2 \theta \, d \theta) \]
\[ = \frac{x^3}{3} - 4x + 8 \theta + C \]
\[ = \frac{x^3}{3} - 4x + 8 \tan^{-1} \left( \frac{x}{2} \right) + C. \]

It is also possible to use the formula \( \int \frac{1}{x^2 + 1} \, dx = \tan^{-1}(x) + C \) by pulling a 4 out of the denominator of \( \frac{1}{x^2 + 4} \) and using an appropriate \( u \)-substitution.

(b) We factor 
\[ x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1). \]
So we make the partial fraction decomposition 
\[ \frac{6x - 4}{x^3 - x} = A \frac{x}{x} + B \frac{1}{x + 1} + C \frac{1}{x - 1}. \]

Adding the right-hand side together we find, 
\[ \frac{6x - 4}{x^3 - x} = \frac{(A + B + C)x^2 + (-B + C)x - A}{x^3 - x}, \]
and so we obtain the system of equations 
\[ A + B + C = 0 \]
\[ -B + C = 6 \]
\[ -A = 4. \]
Solving this system, we find \( A = 4, \, B = -5, \) and \( C = 1. \) Therefore, 
\[ \int \frac{6x - 4}{x^3 - x} \, dx = \left( \frac{4}{x} - \frac{5}{x + 1} + \frac{1}{x - 1} \right) \, dx = 4 \ln |x| - 5 \ln |x + 1| + \ln |x - 1| + C. \]
8. We use separation of variables and find \( \frac{dy}{y} = \frac{-2 \, dx}{\sqrt{1-x^2}} \). Integrating both sides we find
\[
\int \frac{dy}{y} = -2 \int \frac{dx}{\sqrt{1-x^2}}
\]
\[
\ln |y| = -2 \sin^{-1}(x) + C.
\]
Then exponentiating both sides, we find \( y = Ae^{-2 \sin^{-1}(x)} \) for a constant \( A \). It is also possible to solve this equation as a linear first order equation.

9. This is a linear first order equation, so we first divide by \( x^2 \) to put it in the normal form,
\[
\frac{dy}{dx} + \frac{2}{x} y = \frac{\sin(3x)}{x^2}.
\]
The integrating factor \( v(x) = e^{\int \frac{2}{x} \, dx} = e^{2 \ln x} = x^2 \). Therefore, with \( Q(x) = \frac{\sin(3x)}{x^2} \), we have
\[
y = \frac{1}{v(x)} \int v(x)Q(x) \, dx = \frac{1}{x^2} \int x^2 \cdot \frac{\sin(3x)}{x^2} \, dx = \frac{1}{x^2} \int \sin(3x) \, dx
\]
\[
= \frac{1}{x^2} \left[ -\frac{1}{3} \cos(3x) + C \right] = -\frac{\cos(3x)}{3x^2} + C.
\]
Now using the initial condition that \( y(\pi/6) = 2 \), we substitute \( x = \pi/6 \) into this expression to solve for \( C \). We find that \( C = \frac{\pi^2}{18} \) (recalling that \( \cos(\pi/2) = 0 \)). Therefore,
\[
y = -\frac{\cos(3x)}{3x^2} + \frac{\pi^2}{18x^2}.
\]