1. **D**: A simple matter of checking the various formulas reveals that the correct one is \( \frac{(-1)^{n+1}}{n^2 + 1} \).

2. **B**: Because \( 0 \leq \cos^2(n) \leq 1 \) for all \( n \), we have \( 0 \leq \frac{\cos^2(n)}{3} \leq \frac{1}{3} \) for all \( n \). Therefore, we have \( \lim_{n \to \infty} \left( \frac{\cos^2(n)}{3} \right)^n = 0 \), and so
   \[
   \lim_{n \to \infty} \left( \frac{\cos^2(n)}{3} \right)^n + \frac{1}{3} = \frac{1}{3}.
   \]

3. **C**: We break up the series,
   \[
   \sum_{n=0}^{\infty} 2^{2n} - 1 \frac{1}{5^{n+1}} = \sum_{n=0}^{\infty} 2^{2n} - \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{5} \cdot \left( \frac{4}{5} \right)^n - \sum_{n=0}^{\infty} \frac{1}{5} \cdot \left( \frac{1}{5} \right)^n.
   \]
   The two sums on the right are both convergent geometric series, the first with common ratio \( r = \frac{4}{5} \) and the second with common ratio \( r = \frac{1}{5} \). Therefore, using the formula for the sum of a geometric series, the total sum is
   \[
   \sum_{n=0}^{\infty} 2^{2n} - 1 \frac{1}{5^{n+1}} = \frac{1}{1 - \frac{4}{5}} - \frac{1}{1 - \frac{1}{5}} = \frac{3}{4}.
   \]

4. **B**: We want to find \( N \) so that for all \( n > N \), we have \( |a_n - L| < \frac{1}{100} \). First we see that
   \[
   L = \lim_{n \to \infty} \frac{4 - n}{n} = -1.
   \]
   Therefore substituting into the inequality \( |a_n - L| < \frac{1}{100} \), we want \( n \) to satisfy
   \[
   \left| \frac{4 - n}{n} + 1 \right| < \frac{1}{100} \iff \left| \frac{4}{n} \right| < \frac{1}{100}.
   \]
   After cross-multiplying, we see that we need \( n > 400 \). Therefore, \( N = 400 \) and \( N = 1000 \) are both suitable, but \( N = 100 \) is not.

5. **A**: The Alternating Series Estimation Theorem states that the error in summing up the first 10 terms is bounded above by the absolute value of the 11-th term, which is \( \frac{1}{121} \).

6. The series \( \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n + 2} \right) \) is a telescoping series. For part (a), we have the finite sum
   \[
   s_k = \left( 1 - \frac{1}{3} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + \left( \frac{1}{3} - \frac{1}{5} \right) + \left( \frac{1}{4} - \frac{1}{6} \right) + \cdots + \left( \frac{1}{k-1} - \frac{1}{k+1} \right) + \left( \frac{1}{k} - \frac{1}{k+2} \right).
   \]
   After canceling the telescoping terms, we are left with
   \[
   s_k = 1 + \frac{1}{2} - \frac{1}{k+1} - \frac{1}{k+2}.
   \]
   For part (b), we can find the sum of the series by taking the limit,
   \[
   \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n + 2} \right) = \lim_{k \to \infty} s_k = \lim_{k \to \infty} \left( 1 + \frac{1}{2} - \frac{1}{k+1} - \frac{1}{k+2} \right) = 1 + \frac{1}{2} - 0 - 0 = \frac{3}{2}.
   \]
7. (a) \( \lim_{n \to \infty} \frac{3^n}{n^3} = \infty \): This limit can be found in several different ways. One is several quick applications of l'Hôpital's rule (since the form of this limit if \( \frac{\infty}{\infty} \)):

\[
\lim_{n \to \infty} \frac{3^n}{n^3} = \lim_{n \to \infty} \frac{(\ln 3)^3 n}{6n} = \lim_{n \to \infty} \frac{(\ln 3)^3 n}{6} = \infty.
\]

Another way is simply to observe that the numerator \( (3^n) \) grows exponentially, while the denominator \( (n^3) \) is a polynomial. As \( n \to \infty \), the exponential swamps the polynomial, taking the value to \( \infty \).

(b) \( \lim_{n \to \infty} (\ln(3n) - \ln(n - 2)) = \ln 3 \): The familiar properties of logarithms yield,

\[
\lim_{n \to \infty} (\ln(3n) - \ln(n - 2)) = \lim_{n \to \infty} \ln \left( \frac{3n}{n - 2} \right) = \ln(3).
\]

8. Recalling that \( \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \), it follows that \( \frac{1}{1 - x^2} = \sum_{n=0}^{\infty} x^{2n} \). Therefore, if we multiply by \( x \), we find

\[
\frac{x}{1 - x^2} = x \cdot \sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty} x^{2n+1}.
\]

9. (a) This series \( \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{3^n} \) converges. The justification is that since \( |\tan^{-1}(n)| \leq \frac{\pi}{2} \) for all values of \( n \), we have the inequality of series,

\[
\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{3^n} \leq \sum_{n=1}^{\infty} \frac{\pi/2}{3^n}.
\]

The series on the right is a geometric series with \( r = \frac{1}{3} \) and therefore converges. So by the direct comparison test our series converges. It is also possible to use the limit comparison test or the ratio test to show convergence here.

(b) The series \( \sum_{n=1}^{\infty} \cos \left( \frac{1}{n} \right) \) diverges. We calculate the limit of terms,

\[
\lim_{n \to \infty} \cos \left( \frac{1}{n} \right) = \cos(0) = 1 \neq 0,
\]

and so by the test for divergence or \( n \)-th term test, the series diverges. It is also possible to use a comparison test to show divergence here.

(c) The series \( \sum_{n=2}^{\infty} \frac{\sqrt{n^4 - 1}}{n^5} \) converges. Apply the limit comparison test with the series \( \sum_{n=2}^{\infty} \frac{1}{n^3} \):

\[
c = \lim_{n \to \infty} \frac{\sqrt{n^4 - 1}}{n^3} = \lim_{n \to \infty} \sqrt{\frac{n^4 - 1}{n^6}} = \lim_{n \to \infty} \sqrt{\frac{n^4 - 1}{n^2}} = \lim_{n \to \infty} \sqrt{1 - \frac{1}{n^2}} = 1.
\]

Since \( 0 < 1 < \infty \), the limit comparison test says that our series converges if and only if \( \sum_{n=2}^{\infty} \frac{1}{n^3} \) converges. That series converges by the \( p \)-series test with \( p = 3 > 1 \). It is also possible to apply a direct comparison test to show convergence.
10. (a) We use the ratio test,

\[
\rho = \lim_{n \to \infty} \frac{|x-2|}{n+1} \cdot \frac{n^2}{(n+1)^2} = \lim_{n \to \infty} \frac{|x-2|}{n+1} \cdot \frac{n^2}{(n+1)^2} = \lim_{n \to \infty} \frac{n}{n+1} \cdot \frac{|x-2|}{2} = \frac{|x-2|}{2}.
\]

The open interval of convergence can be found by solving for \( \rho < 1 \), so we solve the inequality,

\[
\frac{|x-2|}{2} < 1 \iff |x-2| < 2 \iff -2 < x-2 < 2 \iff 0 < x < 4.
\]

(b) We check the endpoints. At \( x = 0 \), the series is

\[
f(0) = \sum_{n=1}^{\infty} \frac{(-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}.
\]

This sum is the alternating harmonic series, which we know converges. Or this can be shown easily by the alternating series test. At \( x = 4 \), the series is

\[
f(4) = \sum_{n=1}^{\infty} \frac{2^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}.
\]

This series is the harmonic series, which diverges.

(c) We differentiate term-by-term,

\[
f'(x) = \frac{d}{dx} \left( \sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 2^n} \right) = \sum_{n=1}^{\infty} n(x-2)^{n-1} \cdot \frac{1}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{(x-2)^{n-1}}{2^n}.
\]

(d) To calculate \( f'(3) \), we substitute into the series from (c):

\[
f'(3) = \sum_{n=1}^{\infty} \frac{1^{n-1}}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2^n}.
\]

This sum is a geometric series with \( r = \frac{1}{2} \) and first term \( \frac{1}{2} \). Therefore,

\[
f'(3) = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1.
\]