Math 220
Exam #3 Solutions

1. This problem did not cause too much trouble, so I won't draw the graphic for it. If you had problems with it, please let me know.

2. The solutions are as follows: (a) $A = (3,5]$; (b) $B = (-\infty,-4) \cup (4,\infty)$; (c) $A \cap B = (4,5]$; (d) $A - B = (3,4]$; (e) $A \cap (\overline{B} \cup \overline{A}) = (3,4]$.

3. The intervals are $[1,2]$, $[\frac{1}{2},2]$, $[\frac{1}{3},2]$, ..., and of course

$$[1,2] \subseteq [\tfrac{1}{2},2] \subseteq [\tfrac{1}{3},2] \subseteq \cdots \subseteq [\tfrac{1}{i},2] \subseteq \cdots.$$

Thus the answer to (a) is the union of all of these intervals, which is $(0,2]$ (you get as close to 0 as you want to, 0 is never an element of any one of the interavals $[\frac{1}{2},2]$). For (b) the intersection is just the first interval, because each interval is contained in the next and $[1,2]$ is the smallest one, so the answer is $[1,2]$.

4. Proceed as follows:

$$\begin{aligned}
\overline{A \cup (\overline{A} \cap \overline{B})} &= \overline{A} \cap \overline{\overline{A} \cap \overline{B}} && \text{(de Morgan's laws)} \\
&= \overline{A} \cap (A \cup B) && \text{(de Morgan's laws)} \\
&= (\overline{A} \cap A) \cup (\overline{A} \cap B) \\
&= \emptyset \cup (\overline{A} \cap B) \\
&= \overline{A} \cap B \\
&= B - A.
\end{aligned}$$

5. $\{P,N\}$ is <u>not</u> a partition of $\mathbb{R}$. The reason is that $\mathbb{R} \neq P \cup N$, since $0 \notin P$ and $0 \notin N$.

6. The solutions are as follows: (a) $A = [-2,\infty)$; (b) $[1,\infty)$; (c) $[3,4)$.

7. In this problem, we need to show that $C \subseteq A \cup B$ and that $A \cup B \subseteq C$.

<u>$C \subseteq A \cup B$</u>: Let $c \in C$ be arbitrary. Because $c \in C$, we have $h(c) = 0$. By the definition of h then,

$$h(c) = f(c) \cdot g(c) = 0,$$

which implies that $f(c) = 0$ or $g(c) = 0$. If $f(c) = 0$, then $c \in A$ by the definition of A. If $g(c) = 0$, then $c \in B$ by the definition of B. Thus, $c \in A$ or $c \in B$, and so $c \in A \cup B$.

<u>$A \cup B \subseteq C$</u>: Let $c \in A \cup B$ be arbitrary. The first case is $c \in A$: if $c \in A$, then $f(c) = 0$ by definition of A, and so $h(c) = f(c) \cdot g(c) = 0 \cdot g(c) = 0$. The second case is $c \in B$: if $c \in B$, then $g(c) = 0$ by the defintion of B, and so $h(c) = f(c) \cdot g(c) = f(c) \cdot 0 = 0$. In both cases we conclude that $h(c) = 0$, and from the definition of C, we conclude that $c \in C$.

Therefore, $C = A \cup B$.

8. (a) Answers here may vary; (b) Here you can take

$$1 = 5 \cdot 3 + 7 \cdot (-2),$$

or

$$1 = 5 \cdot (-4) + 7 \cdot (3).$$

Other answers are also easy to find.

(c) First of all, $A \subseteq \mathbb{Z}$, since for any integers $s, t \in \mathbb{Z}$ we have $5s + 7t \in \mathbb{Z}$ as well. The hard part is showing that $\mathbb{Z} \subseteq A$. Let $m \in \mathbb{Z}$ be arbitrary. Take any of our equations in (b) and multiply through by m: for example,

$$m = 5 \cdot 3m + 7 \cdot (-2m).$$

Thus if we let $s = 3m$ and $t = -2m$, then $m = 5s + 7t$. That is, $m \in A$. Therefore, $\mathbb{Z} \subseteq A$.