

MATH 220.501
Examination 1
September 25, 2003

NAME _____
SIGNATURE _____

This exam consists of **6** problems, numbered **1–6**. For partial credit you must present your work clearly and understandably.

The point value for each question is shown next to each question.

CHECK THIS EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 6 PROBLEMS ON 6 PAGES (INCLUDING THIS ONE).

Do not mark in the box below.

	Points Possible	Credit
1	15	
2	10	
3	16	
4	15	
5	20	
6	24	
Total	100	

1. [15 points] Determine if the following sentences are **statements**, **open sentences**, or **neither**. If a sentence is open, incorporate it into a **true** statement.

(a) $a^2 + b^2 = c^2$.

(b) Divide both sides of the equation by 10.

(c) It is impossible for a true statement to fail to be false.

(d) The ratio of the circumference to the diameter is π .

(e) For every real number x , if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2. [10 points] Negate the following statements.

(a) There is a $y \in \mathbb{R}$ so that for all $x \in \mathbb{R}$, $(x + y)^2 = x^2 + y^2$.

(b) For all $y \in \mathbb{R}$, there is an $x \in \mathbb{R}$ so that if $y > 0$ then $y = 2^x$.

3. [16 points] Let $A = \{6, 10, 15\}$ and $B = \{3, 5\}$. For each of the following statements, determine if it is **True** or **False**.

(a) $\forall a \in A, \exists b \in B$ so that a is divisible by b . _____

(b) $\exists b \in B$ so that $\forall a \in A, a$ is divisible by b . _____

(c) $\forall b \in B, \exists a \in A$ so that a is divisible by b . _____

(d) $\exists a \in A$ so that $\forall b \in B, a$ is divisible by b . _____

4. [15 points] Starting with the initial symbols provided, translate the given statement into an equivalent statement made of mathematical symbols.

(a) Between any two distinct real numbers there is a third real number.

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}$, if $x < y$, then _____ .

(b) The product of two negative real numbers is positive.

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}$, _____ .

(c) There is no largest integer.

$\forall n \in \mathbb{Z}$, _____ .

5. [20 points] Fill in the blanks of the proof of the following statement:

For every integer n , if n^2 is divided by 4, then the remainder is either 0 or 1.

Proof. Let n _____ .

Case 1: n is even. Since n is even, we can choose $k \in \mathbb{Z}$ so that _____ .

Then

$$n^2 = \text{_____} ,$$

and so $\frac{n^2}{4} = \text{_____}$, and the remainder when n^2 is divided by 4 is _____ .

Case 2: _____ . In this case, we can choose $k \in \mathbb{Z}$ so that

_____ . Then

$$n^2 = \text{_____} ,$$

and so

$$\frac{n^2}{4} = \text{_____} .$$

Thus, in this case, when n^2 is divided by 4, the remainder is _____ .

Therefore, in either case, when n^2 is divided by 4, the remainder is 0 or 1, which is what we wanted to prove.

6. [24 points] Consider the following 11 statement forms.

(A) $P \wedge (P \Rightarrow Q)$

(B) $(\neg Q \Rightarrow \neg P) \wedge (\neg(P \wedge Q))$

(C) $((P \Rightarrow Q) \Rightarrow Q) \Rightarrow P$

(D) $Q \wedge \neg Q$

(E) $P \vee \neg P$

(F) $P \Rightarrow Q$

(G) $Q \Rightarrow P$

(H) $\neg Q$

(I) $\neg P$

(J) $P \wedge Q$

(K) $P \vee Q$

Match up each statement on the left (A–C) with its logically equivalent statement on the right (one of D–K). Explain your answers.

(Bonus) [4 points]

Recall that a prime number is an integer ≥ 2 , whose only positive divisors are itself and 1. Prove the following statement:

For all $n \in \mathbb{N}$, if $2^n - 1$ is a prime number, then n is a prime number.