

MATH 220.501
Examination 3
November 20, 2003

NAME _____
SIGNATURE _____

This exam consists of **8** problems, numbered **1–8**. For partial credit you must present your work clearly and understandably.

The point value for each question is shown next to each question.

CHECK THIS EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 8 PROBLEMS ON 6 PAGES (INCLUDING THIS ONE).

Do not mark in the box below.

	Points Possible	Credit
1	10	
2	15	
3	10	
4	10	
5	10	
6	15	
7	15	
8	15	
Total	100	

1. [10 points] Draw a Venn diagram for sets A , B , C contained in a universe U , and shade in $(A \cup B) - (A \cap C)$.

2. [15 points] Let $A = \{x \in \mathbb{R} \mid -1 < x - 4 \leq 1\}$ and $B = \{x \in \mathbb{R} \mid x^2 > 16\}$. Express the following sets as intervals or unions of intervals.

(a) A

(b) B

(c) $A \cap B$

(d) $A - B$

(e) $A \cap (\overline{B} \cup \overline{A})$

3. [10 points] Express the following sets as intervals or unions of intervals.

(a) $\bigcup_{i=1}^{\infty} [\frac{1}{i}, 2]$

(b) $\bigcap_{i=1}^{\infty} [\frac{1}{i}, 2]$

4. [10 points] Use properties of intersections, unions, and complements to show that for sets A and B in a universe U ,

$$\overline{A \cup (\overline{A} \cap \overline{B})} = B - A.$$

5. [10 points] Let $P = \{x \in \mathbb{R} \mid x > 0\}$ and $N = \{x \in \mathbb{R} \mid x < 0\}$. Is $\{P, N\}$ a partition of \mathbb{R} ? Why or why not?

6. [15 points] Consider the formula $f(x) = 1 + \sqrt{x + 2}$.

(a) What is the largest subset $A \subseteq \mathbb{R}$, so that $f : A \rightarrow \mathbb{R}$ defined by $f(x) = 1 + \sqrt{x + 2}$ is a function?

(a) Compute the image of $f : A \rightarrow \mathbb{R}$.

(c) Compute $f([2, 7))$.

7. [15 points] Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be functions. Define a function $h : \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = f(x) \cdot g(x), \quad x \in \mathbb{R}.$$

Let

$$A = \{x \in \mathbb{R} \mid f(x) = 0\},$$

$$B = \{x \in \mathbb{R} \mid g(x) = 0\},$$

$$C = \{x \in \mathbb{R} \mid h(x) = 0\}.$$

Prove that $C = A \cup B$.

8. [15 points; (a) 3 points; (b) 4 points; (c) 8 points]

Let

$$A = \{5s + 7t \mid s, t \in \mathbb{Z}\}.$$

- (a) List 6 elements of A .

- (b) Show that $1 \in A$.

- (c) Prove that $A = \mathbb{Z}$.