Quiz #3 Solutions

1. (a) Converse: "For all integers m and n, if mn is even, then m is odd and n is even." Contrapositive: "For all integers m and n, if mn is odd, then m is even or n is odd."

   (b) The original statement and its contrapositive are true. The converse is false.

2. Proof: Let a, b, c, and $d \in \mathbb{Z}$ be arbitrary. Suppose that $ab + bc = cd + ba$. Then

$$\begin{aligned} cb + ab &= cd + ab, && \text{(commutativity of } + \text{ and } \cdot) \\ cb &= cd, && \text{(subtracting } ab \text{ from both sides)} \\ cb - cd &= 0, && \text{(subtracting } cd \text{ from both sides)} \\ c(b - d) &= 0. && \text{(distributive law)} \end{aligned}$$

   Since a, b, c, and $d \in \mathbb{Z}$ were arbitrary, it follows that $\forall a, b, c, d \in \mathbb{Z}$, $ab+bc = cd+ba \Rightarrow c(b - d) = 0$.