This exam consists of 8 problems, numbered 1–8. For partial credit you must present your work clearly and understandably and justify your answers.

The use of calculators is not permitted on this exam.

The point value for each question is shown next to each question.

Do not mark in the box below.

<table>
<thead>
<tr>
<th>Points Possible</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
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<tr>
<td>7</td>
<td>20</td>
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<tr>
<td>8</td>
<td>16</td>
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<tr>
<td>Total</td>
<td>75</td>
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</tbody>
</table>
Multiple Choice: [5 points each] In each of Problems 1–5, circle the best answer. There is no partial credit on multiple choice.

1. Suppose $f(x, y)$ is a differentiable function such that it and its derivatives take on the following values:

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$f(x, y)$</th>
<th>$f_x(x, y)$</th>
<th>$f_y(x, y)$</th>
<th>$f_{xx}(x, y)$</th>
<th>$f_{yy}(x, y)$</th>
<th>$f_{xy}(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-2, 2)$</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>$(1, 2)$</td>
<td>0</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>-2</td>
<td>$-\frac{1}{2}$</td>
<td>2</td>
</tr>
<tr>
<td>$(0, 6)$</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>$-\frac{2}{3}$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Which of the following statements is true?

I. $f(x, y)$ has a local minimum at $(x, y) = (-2, 2)$.
II. $f(x, y)$ has a critical point at $(x, y) = (1, 2)$.
III. $f(x, y)$ has a saddle point at $(x, y) = (0, 6)$.

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) II and III only

2. Consider the point $(r, \theta, z) = (2\sqrt{3}, \frac{\pi}{3}, 2)$ in cylindrical coordinates. What are its coordinates in spherical coordinates $(\rho, \theta, \phi)$?

(A) $\left(16, \frac{\pi}{3}, \frac{\pi}{6}\right)$
(B) $\left(16, \frac{\pi}{2}, \frac{\pi}{4}\right)$
(C) $\left(4, \frac{\pi}{3}, \frac{\pi}{2}\right)$
(D) $\left(4, \frac{\pi}{3}, \frac{\pi}{6}\right)$
(E) $\left(4, \frac{\pi}{3}, \frac{\pi}{4}\right)$

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3. If we change the order of integration on the integral 
\[ \int_{0}^{3} \int_{1}^{\sqrt{x+1}} f(x, y) \, dy \, dx \], we obtain which integral below?

(A) \[ \int_{1}^{3} \int_{1}^{y^2+1} f(x, y) \, dx \, dy \]

(B) \[ \int_{0}^{2} \int_{0}^{y^2-1} f(x, y) \, dx \, dy \]

(C) \[ \int_{1}^{2} \int_{0}^{y^2-1} f(x, y) \, dx \, dy \]

(D) \[ \int_{0}^{2} \int_{y^2-1}^{1} f(x, y) \, dx \, dy \]

(E) \[ \int_{1}^{2} \int_{y^2-1}^{3} f(x, y) \, dx \, dy \]

4. Let D be the disk of radius 2 centered at \((x, y) = (2, 0)\), and let R be the upper half of D. Suppose that R is given the mass density \(\rho(x, y)\) defined by

\[ \rho(x, y) = \text{distance from (x, y) to the origin} \]

Supposer that \(m\) is the mass of R. Which integral below gives the x-coordinate of the center of mass of R?

(A) \[ \frac{1}{m} \int_{0}^{\pi} \int_{0}^{4 \sin \theta} r^2 \cos \theta \, dr \, d\theta \]

(B) \[ \frac{1}{m} \int_{0}^{\pi} \int_{0}^{4 \cos \theta} r^3 \cos \theta \, dr \, d\theta \]

(C) \[ \frac{1}{m} \int_{0}^{\pi} \int_{0}^{2} r^3 \sin \theta \, dr \, d\theta \]

(D) \[ \frac{1}{m} \int_{0}^{4} \int_{0}^{\sqrt{4-x^2}} x \sqrt{x^2 + y^2} \, dy \, dx \]

(E) \[ \frac{1}{m} \int_{0}^{4} \int_{0}^{\sqrt{4-(x-2)^2}} \sqrt{x^2 + y^2} \, dy \, dx \]
5. Let $E$ be the region in the first octant bounded below by the plane $z = 5$ and above by the paraboloid $z = 9 - x^2 - y^2$. Which integral below expresses the volume of $E$?

(A) $\int_{0}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{5}^{9-x^2-y^2} dz \, dy \, dx$

(B) $\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{0}^{9-x^2-y^2} dz \, dx \, dy$

(C) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{9-x^2-y^2} dz \, dy \, dx$

(D) $\int_{0}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{5}^{9-x^2-y^2} dz \, dx \, dy$

(E) none of the above

6. [14 points] Let $f(x, y) = xy + 1$. What are the maximum and minimum values attained by $f(x, y)$ on the ellipse $x^2 + 2y^2 = 16$? At what points $(x, y)$ do they occur? You must use the method of Lagrange multipliers.
7. [(a) 8 points; (b) 12 points] Let $R$ be the first quadrant sector (pie wedge) of the circle $x^2 + y^2 = 4$ between $x = 0$ and $y = x$. Let $E$ be the solid in 3-space that is above $R$ and below the surface $z = \sqrt{9 + x^2 + y^2}$.

(a) Express the volume of $E$ as an iterated double integral in rectangular coordinates. (You need not evaluate the integral).

(b) Convert the integral from part (a) into an iterated double integral in polar coordinates and evaluate it.
8. [16 points] Let $F$ be the solid in 3-space that is above the $xy$-plane and between the sphere $x^2 + y^2 + z^2 = 18$ and the cone $z^2 = x^2 + y^2$. Suppose also that $F$ has mass density $d(x, y) = x^2 + y^2$.

(a) Express the volume of $F$ as an iterated triple integral in spherical coordinates. (You need not evaluate the integral.)

(b) Express the mass of $F$ as an iterated triple integral in cylindrical coordinates. (You need not evaluate the integral.)