1. Consider the point \((r, \theta, z) = (2\sqrt{3}, \frac{\pi}{3}, 2)\) in cylindrical coordinates. What are its coordinates in spherical coordinates \((\rho, \theta, \phi)\)?

2. Let \(E\) be the region in the first octant bounded below by the plane \(z = 5\) and above by the paraboloid \(z = 9 - x^2 - y^2\). Write down an iterated triple integral in rectangular coordinates that expresses the volume of \(E\).

3. Let \(F\) be the solid in 3-space that is above the \(xy\)-plane and between the sphere \(x^2 + y^2 + z^2 = 18\) and the cone \(z^2 = x^2 + y^2\). Suppose also that \(F\) has mass density \(d(x, y) = x^2 + y^2\).

   (a) Express the volume of \(F\) as an iterated triple integral in spherical coordinates. (You need not evaluate the integral.)

   (b) Express the mass of \(F\) as an iterated triple integral in cylindrical coordinates. (You need not evaluate the integral.)

4. Let \(f(x, y, z) = z \sin(xy)\). Find \(\text{div}(\nabla f)\).

5. Let \(C\) be the curve in the \(xy\)-plane given by \(x = 3t, y = 2t^2, 0 \leq t \leq 1\). Evaluate \(\int_C 3y\, dx - x^2\, dy\).

6. Let \(C\) be the curve defined by parametric equations \(x = -t \cos t, y = t \sin t, z = te^{t/\pi}\) with \(0 \leq t \leq \pi\). Let \(F(x, y, z) = (1 + yz, 1 + xz, 1 + xy)\). Compute \(\int_C F \cdot dr\).

   Hint: \(F\) is a conservative vector field—find a potential function for \(F\) and use independence of path.

7. Let \(S\) be the portion of the paraboloid \(z = 1 - x^2 - y^2\) above the \(xy\)-plane, and let \(n\) be an upward unit normal vector for \(S\). Let \(F(x, y, z) = (x, y, 2z)\). Use the Divergence Theorem to find the value of the surface integral \(\iint_S F \cdot n\, dS\).

8. Evaluate the following line integrals.

   (a) Evaluate \(\int_C (x^2 + y^2 + z^2)\, ds\), where \(C\) is the curve in 3-space given by \(x = 4 \cos t, y = 4 \sin t, z = 3t, 0 \leq t \leq 2\pi\).

   (b) Evaluate \(\int_C (3ye^{x^2} + 2e^x)\, dx + \left(\frac{3}{2}x^2 + \sin y\right)\, dy\), where \(C\) is the triangle in the \(xy\)-plane with vertices \((0, 0), (2, 0), (2, 4)\), oriented counter-clockwise. (Hint: Use Green’s Theorem.)
9. Let $\mathbf{F} = 2y\mathbf{i} - 2x\mathbf{j}$ be a vector field in the $xy$-plane. Let $C$ be the unit circle oriented counterclockwise, and let $R$ be the closed unit disk.

(a) According to Green’s Theorem, $\oint_C \mathbf{F} \cdot d\mathbf{r}$ is equal to what iterated double integral in rectangular coordinates?

(b) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ directly.

(c) Evaluate the double integral you provided in part (a) directly.

10. Use Stokes’ Theorem to calculate the surface integral $\iint_S (\text{curl} \mathbf{F}) \cdot \mathbf{n} \, dS$, where $S$ is the part of the paraboloid $z = 9 - x^2 - y^2$ above the $xy$-plane, $\mathbf{n}$ is the outward unit normal, and $\mathbf{F} = \langle z + y, z - x, 2z \rangle$.

11. Let $S$ be the graph of $f(x, y) = x^2 + y^2$ above the unit disk $R$ in the $xy$-plane. The boundary of $S$ is the circle $C$ that is a translate of the usual unit circle in the $xy$-plane, 1 unit directly upward.

Let $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + yz\mathbf{k}$. Use Stokes’ Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is oriented counter-clockwise.