

MATH 251.509
Examination 1
February 23, 2006

NAME _____
SIGNATURE _____

This exam consists of **10** problems, numbered **1–10**. For partial credit you must present your work clearly and understandably and justify your answers.

The use of calculators is not permitted on this exam.

The point value for each question is shown next to each question.

CHECK THIS EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 10 PROBLEMS ON 6 PAGES (INCLUDING THIS ONE).

Do not mark in the box below.

	Points Possible	Credit
1–6	42	
7	10	
8	10	
9	18	
10	20	
Total	100	

Multiple Choice: [7 points each] In each of Problems 1–6, circle the best answer.

1. Let L be the line given by parametric equations $x = 6 + 7t$, $y = -3 - 4t$, $z = 5 - 2t$. What is the equation of the plane that contains the point $(3, -5, 2)$ and is perpendicular to L ?
 - (A) $6(x - 3) - 3(y + 5) + 5(z - 2) = 0$
 - (B) $6(x + 3) - 3(y - 5) + 5(z + 2) = 0$
 - (C) $7(x - 3) - 4(y + 5) - 2(z - 2) = 0$
 - (D) $7(x + 3) - 4(y - 5) - 2(z + 2) = 0$
 - (E) $3(x - 7) - 5(y + 4) + 2(z + 2) = 0$

2. Let $f(x, y) = xy^2 \cos(x^2 - 3y)$. What is $\frac{\partial f}{\partial y}$?
 - (A) $-2xy \sin(x^2 - 3y)$
 - (B) $6xy \sin(x^2 - 3y)$
 - (C) $6xy \sin(x^2 - 3y) - 2xy^2 \sin(x^2 - 3y)$
 - (D) $2xy \cos(x^2 - 3y) + 3xy^2 \sin(x^2 - 3y)$
 - (E) $2xy \cos(x^2 - 3y) - 2x^2y^2 \sin(x^2 - 3y)$

3. Let $g(x, y) = \frac{\sin(\sqrt{1 - x - y})}{x - y}$. What is the domain of g ?
 - (A) $(-\infty, \infty)$
 - (B) $\{(x, y) \mid x \geq y\}$
 - (C) $\{(x, y) \mid x \neq y\}$
 - (D) $\{(x, y) \mid x + y \leq 1\}$
 - (E) $\{(x, y) \mid x + y \leq 1 \text{ and } x \neq y\}$

4. Consider the following surface in 3-space,

$$S : z - x^2 = y^2 + 25.$$

What type of surface is S ?

- (A) sphere
 - (B) cylinder
 - (C) ellipsoid
 - (D) hyperboloid
 - (E) paraboloid
5. Let $\mathbf{a} = \langle 3, 5, -2 \rangle$ and $\mathbf{b} = \langle 6, -4, -1 \rangle$. Consider the statements:

- I. \mathbf{a} is parallel \mathbf{b} .
- II. \mathbf{a} is perpendicular to \mathbf{b} .
- III. \mathbf{a} is perpendicular to $\mathbf{a} \times \mathbf{b}$.

Which of these statements are true?

- (A) I and III only
 - (B) II and III only
 - (C) I only
 - (D) II only
 - (E) III only
6. Calculate the value of the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^2 - 2x}{3x - x^2y}$.
- (A) 0
 - (B) 1
 - (C) -1
 - (D) $-\frac{2}{3}$
 - (E) Does not exist

7. [10 points] Calculate $\frac{\partial^2 g}{\partial x \partial y}$ if $g(x, y) = 3x^2y \ln(x - y)$.

8. [10 points] Suppose $f(x, y)$ is a differentiable function. Let

$$g(t, u) = f(t + 3u, te^u).$$

Find $\frac{\partial g}{\partial t}$ and $\frac{\partial g}{\partial u}$ in terms of t , u , $\frac{\partial f}{\partial x}$, and $\frac{\partial f}{\partial y}$.

9. [18 points] Consider the three points $P = (2, 0, 1)$, $Q = (-2, 2, 0)$, $R = (0, 5, -3)$.

(a) What is the length of the vector \overrightarrow{QR} ?

(b) What is the area of the triangle $\triangle PQR$?

(c) Is angle $\angle PQR$ an obtuse angle? Why or why not?

10. [20 points] Let $f(x, y) = 3 + x^2 - 5y + xy - y^2$.

(a) Find the equation of the tangent plane to the graph of $f(x, y)$ above the point $(x, y) = (2, -1)$.

(b) At what points on the graph of $f(x, y)$ is the tangent plane parallel to the xy -plane?