MATH 251.509
Examination 2
April 11, 2006

NAME ______________________________
SIGNATURE __________________________

This exam consists of **11** problems, numbered **1–11**. For partial credit you must present your work clearly and understandably and justify your answers.

The use of calculators is not permitted on this exam.

The point value for each question is shown next to each question.

CHECK THIS EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 11 PROBLEMS ON 8 PAGES (INCLUDING THIS ONE).

Do not mark in the box below.

	Points Possible	Credit
1–7	42	
8	15	
9	15	
10	14	
11	14	
Total	100	

Multiple Choice: [6 points each] In each of Problems 1–7, circle the best answer.

1. Let $f(x, y)$ be a differentiable function on the closed disk D of radius 4 centered at $(0, 0)$ in the xy-plane. Suppose we know the following information:

   - $f(x, y)$ has critical points at $(x, y) = (1, 1)$, $(x, y) = (2, 3)$, and $(x, y) = (-1, 2)$;
   - $f(1, 1) = 3$, $f(2, 3) = -5$, and $f(-1, 2) = 1$;
   - $f(x, y) = 0$ for all (x, y) on the boundary of D.

   Consider the statements:

   I. The absolute maximum value of $f(x, y)$ on D is 3.
   II. $f(x, y)$ has a saddle point at $(x, y) = (-1, 2)$.
   III. $f(x, y)$ has a local minimum at $(x, y) = (2, 3)$.

   Which of these statements must be true?

   (A) I only

   (B) II only

   (C) I and II only

   (D) I and III only

   (E) I, II, and III

2. What is the equation of the tangent plane to the graph of $e^{2x}\cos(yz) + 2x^2 - z^3 = 0$ at the point $(0, 0, 1)$?

   (A) $x + y + z + 3 = 0$

   (B) $2x - 3z + 3 = 0$

   (C) $x - 3y + 1 = 0$

   (D) $2x - y + z + 1 = 0$

   (E) $2x - y - 3z + 3 = 0$

3. Let R be the region in the xy-plane bounded by $y = x^2 - 1$ and $y = x + 5$, and let m be its mass. If R has a mass density per unit area given by the function $\rho(x, y) = e^{xy}$, which iterated integral below gives the x-coordinate of the center of mass of R?

(A) $\dfrac{1}{m} \displaystyle\int_{-2}^{3} \int_{x^2-1}^{x+5} xe^{xy}\, dy\, dx$

(B) $\dfrac{1}{m} \displaystyle\int_{-2}^{3} \int_{x+5}^{x^2-1} yx^{xy}\, dy\, dx$

(C) $\dfrac{1}{m} \displaystyle\int_{-3}^{2} \int_{0}^{x^2-1} xe^{xy}\, dy\, dx$

(D) $\dfrac{1}{m} \displaystyle\int_{-3}^{2} \int_{x^2-1}^{x+5} e^{xy}\, dy\, dx$

(E) $\dfrac{1}{m} \displaystyle\int_{2}^{3} \int_{x^2-1}^{x+5} xe^{xy}\, dy\, dx$

4. What are the points in polar coordinates (r, θ) where the two circles $r = \sqrt{3}$ and $r = 2\cos\theta$ intersect?

(A) $(\sqrt{3}, \frac{\pi}{2})$ and $(-\sqrt{3}, \frac{3\pi}{2})$

(B) $(\sqrt{3}, \frac{\pi}{3})$ and $(-\sqrt{3}, \frac{2\pi}{3})$

(C) $(\sqrt{3}, \frac{\pi}{4})$ and $(-\sqrt{3}, \frac{3\pi}{4})$

(D) $(\sqrt{3}, \frac{\pi}{6})$ and $(-\sqrt{3}, \frac{5\pi}{6})$

(E) $(\sqrt{3}, 0)$ and $(-\sqrt{3}, \pi)$

5. The point P has spherical coordinates $(\rho, \theta, \phi) = (2, \frac{\pi}{4}, \frac{\pi}{6})$. What are its cylindrical coordinates (r, θ, z)?

(A) $(1, \frac{\pi}{4}, 2)$

(B) $(2, \sqrt{2}, -\frac{\pi}{6})$

(C) $(4, \frac{\pi}{4}, \sqrt{3})$

(D) $(\sqrt{2}, \frac{\pi}{4}, \sqrt{3})$

(E) $(1, \frac{\pi}{4}, \sqrt{3})$

6. If we change the order of integration on the integral $\int_0^3 \int_0^{y^2} f(x,y)\,dx\,dy$ we obtain which integral below?

(A) $\int_0^{\sqrt{3}} \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

(B) $\int_0^{9} \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

(C) $\int_0^{9} \int_{\sqrt{x}}^{3} f(x,y)\,dy\,dx$

(D) $\int_0^{\sqrt{3}} \int_{\sqrt{x}}^{3} f(x,y)\,dy\,dx$

(E) $\int_0^{3} \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

7. Let T be the tetrahedron in the first octant whose vertices are the four points $(0,0,0)$, $(2,0,0)$, $(0,3,0)$, and $(0,0,6)$. Which integral below expresses the volume of V?

(A) $\int_0^{3} \int_0^{2} \int_0^{6} dz\,dx\,dy$

(B) $\int_0^{3} \int_0^{\frac{2}{3}y+2} \int_0^{6-3x-2y} dz\,dx\,dy$

(C) $\int_0^{2} \int_0^{-\frac{3}{2}x+3} \int_0^{6-3x-2y} dz\,dy\,dx$

(D) $\int_0^{6} \int_0^{-\frac{1}{2}z+2} \int_0^{6-3x-z} dy\,dx\,dz$

(E) None of the above.

8. [15 points] Let

$$f(x, y) = xy - \frac{1}{3}x^3 - y^2.$$

Find all of the critical points of $f(x, y)$ and classify each of them as either a *local maximum*, *local minimum*, or *saddle point*. Justify your answers.

9. [15 points] Let T be the solid in 3-space bounded above by the paraboloid $z = 12 - 2x^2 - 2y^2$ and below by the plane $z = 4$.

(a) Express the volume of T as an iterated double integral in rectangular coordinates.

(b) Express the volume of T as an iterated double integral in polar coordinates.

(c) Find V.

10. [14 points] Consider $\int_0^2 \int_{2x}^4 \frac{1}{\sqrt{1+y^2}}\, dy\, dx.$

(a) Sketch the region of integration in the xy-plane for the integral above. Be sure to label the parts of your sketch.

(b) Evaluate the integral.

11. [14 points] Let E be the solid in 3-space bounded above by the cone $z = \sqrt{x^2 + y^2}$ and below by the paraboloid $z = x^2 + y^2$. Suppose that E has a mass density of $\delta(x, y, z) = 3xyz$.

(a) Express the <u>volume</u> of E as a triple integral in rectangular coordinates. (You need not evaluate the integral.)

(b) Express the <u>mass</u> of E as a triple integral in cylindrical coordinates. (You need not evaluate the integral.)