

Math 304
Exam 1 Sample Problems
February 15, 2004

The exam will cover material from class, the homework, and the reading. Included below are some sample problems, which should give you a sampling of what types of things to expect on the exam. One thing though, this is not meant to be an exhaustive list, and there may be types of problems on the exam that differ from these.

1. Consider the systems of equations,

$$\begin{aligned}3s - 4u &= -1 \\s + t + u &= 0 \\2s + t - u &= -3,\end{aligned}$$

and

$$\begin{aligned}x + y + z + v + w &= 7 \\x + y - z + 2v &= 11 \\-x - y - z + v + w &= 13.\end{aligned}$$

- (a) Convert each system of equations to an equivalent matrix equation $A\mathbf{x} = \mathbf{b}$, and write its augmented matrix $[A \mid \mathbf{b}]$.
- (b) Carry out on paper, step by step, the row reduction algorithm on each matrix to transform it into reduced row echelon form.
- (c) Describe completely the set of solutions of each system of equations.
2. Take the determinants of the following matrices. Be sure to show your work. Which ones are invertible and why?

(a) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{bmatrix}$.

3. Carry out on paper, step by step, the process of finding the inverses of the invertible matrices in Problem #2.

4. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 1 & -2 & 1 \\ 1 & -3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix}.$$

(a) Find elementary matrices E and F so that $AE = B$ and $FB = C$.

(b) Find elementary matrices E_1, \dots, E_k so that $(E_k \cdot E_{k-1} \cdots E_2 E_1)C = I$.

5. Let

$$A = \begin{bmatrix} -1 & 0 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}.$$

If n is a positive integer, what is $(AB)^n$?

6. Suppose A is a 2×2 matrix with the property that $AA^T = I$. Show that $\det(A) = \pm 1$.

7. Calculate the determinant of the following matrix.

$$A = \begin{bmatrix} \cos(y) & 0 & \sin(y) \\ \sin(x) \sin(y) & \cos(x) & -\sin(x) \cos(y) \\ -\cos(x) \cos(y) & \sin(x) & \cos(x) \cos(y) \end{bmatrix}$$

Additional Problems from Leon:

- 1.1: 6h, 10
- 1.2: 6d, 9, 10
- 1.3: 10, 14b, 23
- 1.4: 2, 8, 14
- 1.5: 1de, 13
- 2.1: problems in HW
- 2.2: problems in HW