

1. [10 points] Form the augmented matrix of the system of equations,

$$\begin{aligned}3x_1 + 2x_2 + 3x_3 - 2x_4 &= 1 \\x_1 + x_2 + x_3 &= 3 \\x_1 + 2x_2 + x_3 - x_4 &= 2,\end{aligned}$$

and then use elementary row operations to put it in reduced row echelon form.

*Solution:* The augmented matrix is

$$\left[ \begin{array}{cccc|c} 3 & 2 & 3 & -2 & 1 \\ 1 & 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & -1 & 2 \end{array} \right].$$

To put it into reduced row echelon form, the best thing to do is start by swapping out the first row so that you don't have to immediately divide the first row by 3. After performing the necessary row operations you obtain the following matrix in reduced row echelon form:

$$\left[ \begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

2. [10 points] Consider the augmented matrix

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -2 & 3 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

What are the leading and free variables for the associated system of equations? What is the solution set of the associated system of equations?

*Solution:* Taking the columns to correspond to  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ , and  $x_5$ , the leading variables are  $x_1$ ,  $x_2$ , and  $x_4$ ; the free variables are  $x_3$  and  $x_5$ . We see that

$$\begin{aligned}x_1 &= -2x_3 + 2x_5 + 3 \\x_2 &= x_3 - x_5 + 1 \\x_4 &= 2x_5 + 2.\end{aligned}$$

So we see that the solution set (setting  $\alpha = x_3$  and  $\beta = x_5$ ) for our system of equations is

$$\{(-2\alpha + 2\beta + 2, \alpha - \beta + 1, \alpha, 2\beta + 2, \beta) \mid \alpha, \beta \in \mathbb{R}\}.$$