

1. [10 points] Let

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}.$$

Write down the elementary matrices that (a) swap the 1st and 3rd rows of A and (b) replace the 2nd row with its sum with twice the 1st row.

$$\text{Solution: (a) } \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(b) } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

2. [10 points] Let

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}.$$

(a) Find a 2×2 matrix B so that $AB = O$ but $BA \neq O$, where $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. (b) Is A invertible? Why or why not?

Solution: (a) There are many choices for B . One such choice is

$$B = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}.$$

We see that

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

but

$$BA = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}.$$

(b) A is not invertible. One way to see this is that A is already in reduced row echelon form, so in particular we will not be able to row reduce it to the identity matrix.