

**Math 323**  
**Exam 1 Sample Problems**  
September 27, 2013

The exam will cover material from class, the homework, and the reading. Included below are some sample problems, which should give you a sampling of what types of things to expect on the exam. However, this is not meant to be an exhaustive list, and there may be types of problems on the exam that differ from these.

1. Consider the systems of equations,

$$\begin{aligned}3s - 4u &= -1 \\s + t + u &= 0 \\2s + t - u &= -3,\end{aligned}$$

and

$$\begin{aligned}x + y + z + v + w &= 7 \\x + y - z + 2v &= 11 \\-x - y - z + v + w &= 13.\end{aligned}$$

- (a) Convert each system of equations to an equivalent matrix equation  $A\mathbf{x} = \mathbf{b}$ , and write its augmented matrix  $[A \mid \mathbf{b}]$ .
- (b) Carry out on paper, step by step, the row reduction algorithm on each matrix to transform it into reduced row echelon form.
- (c) Describe completely the set of solutions of each system of equations.
2. Take the determinants of the following matrices. Be sure to show your work. Which ones are invertible and why?

(a)  $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & 3 & -1 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{bmatrix}$ .

3. Carry out on paper, step by step, the process of finding the inverses of the invertible matrices in Problem #2.

4. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 1 & -2 & 1 \\ 1 & -3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -2 & -2 \\ 1 & -3 & 1 \end{bmatrix}.$$

- (a) Find elementary matrices  $E$  and  $F$  so that  $AE = B$  and  $FB = C$ .  
(b) Find elementary matrices  $E_1, \dots, E_k$  so that  $(E_k \cdot E_{k-1} \cdots E_2 E_1)C = I$ .

5. Let

$$A = \begin{bmatrix} -1 & 0 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}.$$

If  $n$  is a positive integer, what is  $(AB)^n$ ?

6. Suppose  $A$  is a  $2 \times 2$  matrix with the property that  $AA^T = I$ . Show that  $\det(A) = \pm 1$ .

7. Calculate the determinant of the following matrix.

$$A = \begin{bmatrix} \cos(y) & 0 & \sin(y) \\ \sin(x)\sin(y) & \cos(x) & -\sin(x)\cos(y) \\ -\cos(x)\sin(y) & \sin(x) & \cos(x)\cos(y) \end{bmatrix}$$

8. Determine if the following sets of vectors are or are not vector spaces. If they are not, explain why.

- (a)  $V =$  solution set of the equations  $x + y - z - w = 0$  and  $x + y + 2w = 0$  in  $\mathbb{R}^4$ .  
(b)  $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid y = x + \frac{1}{x} \right\}$ .  
(c)  $X =$  set of upper triangular  $3 \times 3$  matrices.

9. Let

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

- (a) Show that  $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$  is in the nullspace  $N(A)$  of  $A$ .  
(b) Find linear equations that define  $N(A)$  as a subset of  $\mathbb{R}^4$ .  
(c) Find a set of vectors that span  $N(A)$ .

10. Let  $A$  and  $B$  be  $m \times n$  matrices. Prove that  $A$  is row equivalent to  $B$  if and only if  $B$  is row equivalent to  $A$ .

11. Let  $A$  and  $B$  be  $k \times k$  matrices, and let

$$M = \begin{bmatrix} 0 & B \\ A & 0 \end{bmatrix}.$$

Prove that  $\det(M) = (-1)^k \det(A) \det(B)$ .

12. Prove that if  $S$  is subspace of  $\mathbb{R}^1$ , then either  $S = \{0\}$  or  $S = \mathbb{R}^1$ .