Math 323
Final Exam Sample Problems
December 7, 2013

The exam will cover material from class, the homework, and the reading. Included below are some sample problems, which should give you a sampling of what types of things to expect on the exam. One thing though, this is not meant to be an exhaustive list, and there may be types of problems on the exam that differ from these. These problems only cover the material since the second exam.

1. Let
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{pmatrix}.$$
   (a) Find the eigenvalues of A.
   (b) For each eigenvalue, find a basis for the corresponding eigenspace.
   (c) Find a matrix Q so that $A = QDQ^{-1}$, where D is a diagonal matrix.
   (d) Use the decomposition in (c) to compute A^5.

2. Let A be a nonsingular $n \times n$ matrix, and let λ be an eigenvalue of A.
   (a) Prove that $\lambda \neq 0$.
   (b) Prove that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

3. Let
$$A = \begin{pmatrix} 4 & 1 & 2 & 3 \\ -2 & 3 & 1 & 4 \end{pmatrix}.$$
   Find bases for the following spaces.
   (a) The null space $N(A)$.
   (b) The orthogonal complement $N(A)^{\perp}$.

4. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthonormal basis for a 3-dimensional subspace S of an inner product space V, and let $\mathbf{x} = 2\mathbf{u}_1 - 2\mathbf{u}_2 + \mathbf{u}_3$ and $\mathbf{y} = 3\mathbf{u}_1 + \mathbf{u}_2 - 4\mathbf{u}_3$.
   (a) Determine the value of $\langle \mathbf{x}, \mathbf{y} \rangle$.
   (b) Determine the value of $\|\mathbf{x}\|$.

5. Let $V = C[-\pi, \pi]$ be the space of continuous functions on the interval $[\pi, \pi]$ with inner product
$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)\, dx.$$
   Take as given that $\{\frac{1}{\sqrt{2}}, \cos(x), \cos(2x)\}$ is an orthonormal set in V.
   (a) Show that $\cos(x)$ and $\sin(x)$ are orthogonal in V.
   (b) Given that $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, find $\langle \sin^2(x), \frac{1}{\sqrt{2}} \rangle$, $\langle \sin^2(x), \cos(x) \rangle$, and $\langle \sin^2(x), \cos(2x) \rangle$, without computing the integrals directly.

6. Leon p. 268, problem #1.