# Math 415
# HW #9 Hints/Solutions
# November 16, 2004

In the problems that are to be turned in for a grade (the '*' problems), I give a hint. For the other problems I explain how to solve the problem, though a proper write-up will need more details and explanation.

Section 14, #10. Since $|\mathbb{Z}_{60}| = 60$ and $|\langle 12\rangle| = 5$, it follows from Lagrange's Theorem that $\mathbb{Z}_{60}/\langle 12\rangle$ has $60/5 = 12$ elements. It is not hard to check that $26 + \langle 12\rangle$ has order 6, since $6 \cdot 26$ is divisible by 12.

16. Hint: Once you wind through all of the notation, this isn't too bad. The group $i_{\rho_0}[H]$ will be a subgroup of order 2 in $S_3$.

23. (f), (h), and (j) are False. Rest are True.

24. Hint: Normality is not not so bad. To describe the quotient $S_n/A_n$, first determine $|S_n/A_n|$. What is the only group of that order up to isomorphism?

28. This problem, like #23 requires finding all the subgroups of $\mathbb{Z}_{20}$. We did this in class on Monday!

27. The thing here is to get the right equivalence relation. We let $S$ = set of all subgroups of $G$. Then for $H$ and $K \in S$, we say "$H$ is conjugate to $K$" if and only if there exists a $g \in G$ so that $gHg^{-1} = K$. (That is, we need just 1 such $g$ for them to be conjugate.) Reflexivity: Since $eHe^{-1} = H$, we see that $H$ is conjugate to $H$. Symmetry: If $H$ is conjugate to $K$, then we can pick a $g \in G$ so that $gHg^{-1} = K$. Then we check that $g^{-1}Kg = H$. Since $g^{-1}Kg = g^{-1}K(g^{-1})^{-1}$, we see that $K$ is conjugate to $H$. Transitivity is similar.

29. Hint: You just slog it out. Note that any two conjugate subgroups are isomorphic and so must have the same order.

30. Hint: Use exercise 40 on p. 103.

38. Hint: Let $H = \{g \in G \mid \forall x \in G,\ i_g(x) = x\}$. The problem is asking you to show that $H$ is a normal subgroup of $G$.

Section 15, #13. Hint: Check the answer in the back of the book....

14. The center is $\mathbb{Z}_3 \times \{e\} = \{(0,e),(1,e),(2,e)\}$. The commutator is $\{0\} \times \langle\sigma\rangle = \{(0,e),(0,\sigma),(0,\sigma^2)\}$.

15. Hint: The center of $S_3 \times D_4$ is (the center of $S_3$) $\times$ (the center of $D_4$). The commutator is more tricky.

34. Hint: Let $H$ be a non-trivial subgroup of index 2 in $G$. Then $|H| = \frac{1}{2}|G|$. Show that $H$ must be normal in $G$.

35. Hint: Use the multiplicative property of homomorphisms. It goes fairly smoothly.

37. Follow their hint. Keep in mind that you want to use the assumption that $G/Z(G)$ is cyclic to prove that $G$ is abelian.